VECTORS WITH VIDEO GAMES

Will Monroe
Splash! Teaching Program
April 11-12, 2015

Image credit: Wikimedia Commons user Manop

Video: Portal clip

Image credit: DGBthekafu / Wikimedia Commons user Bouncey2k

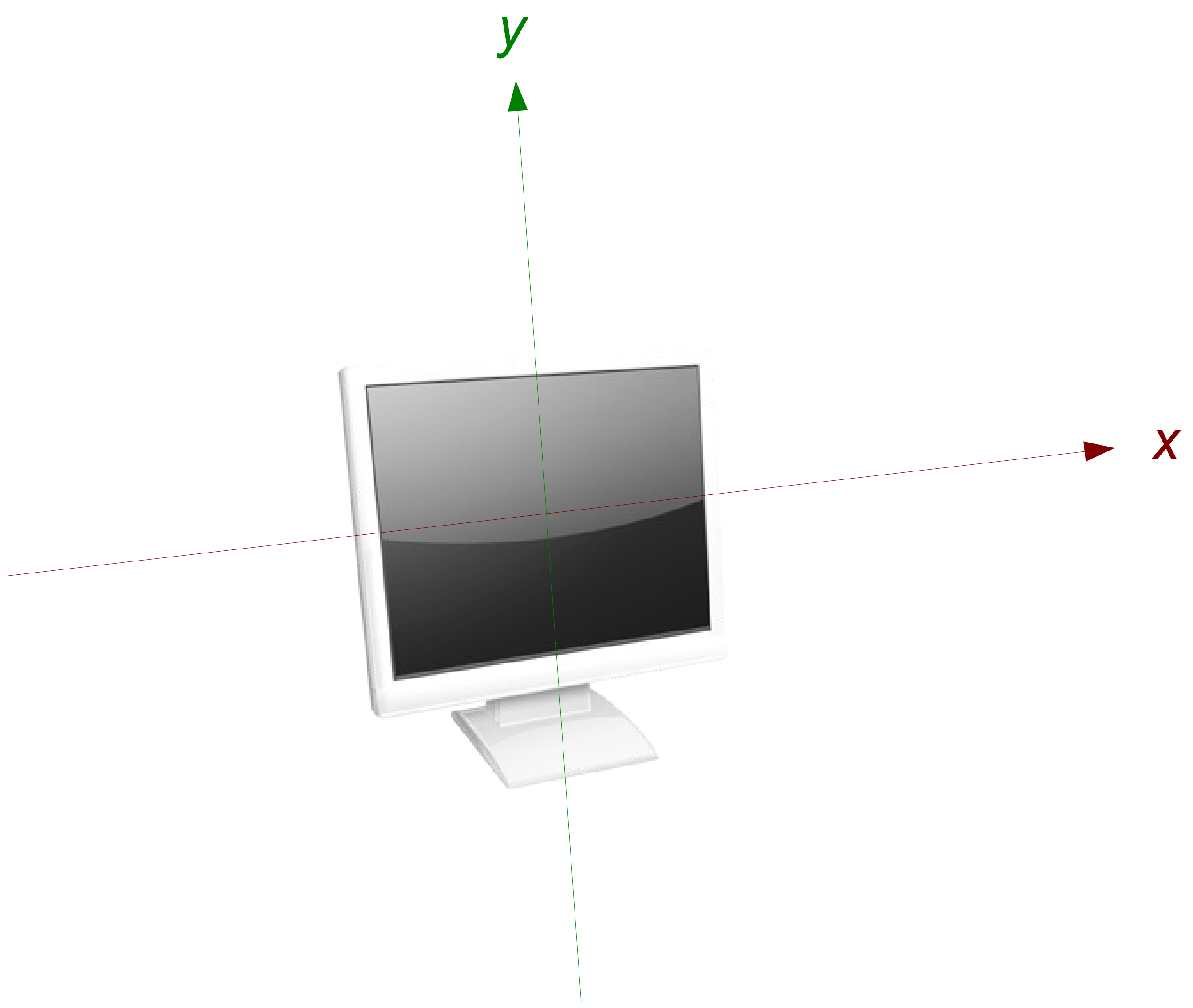
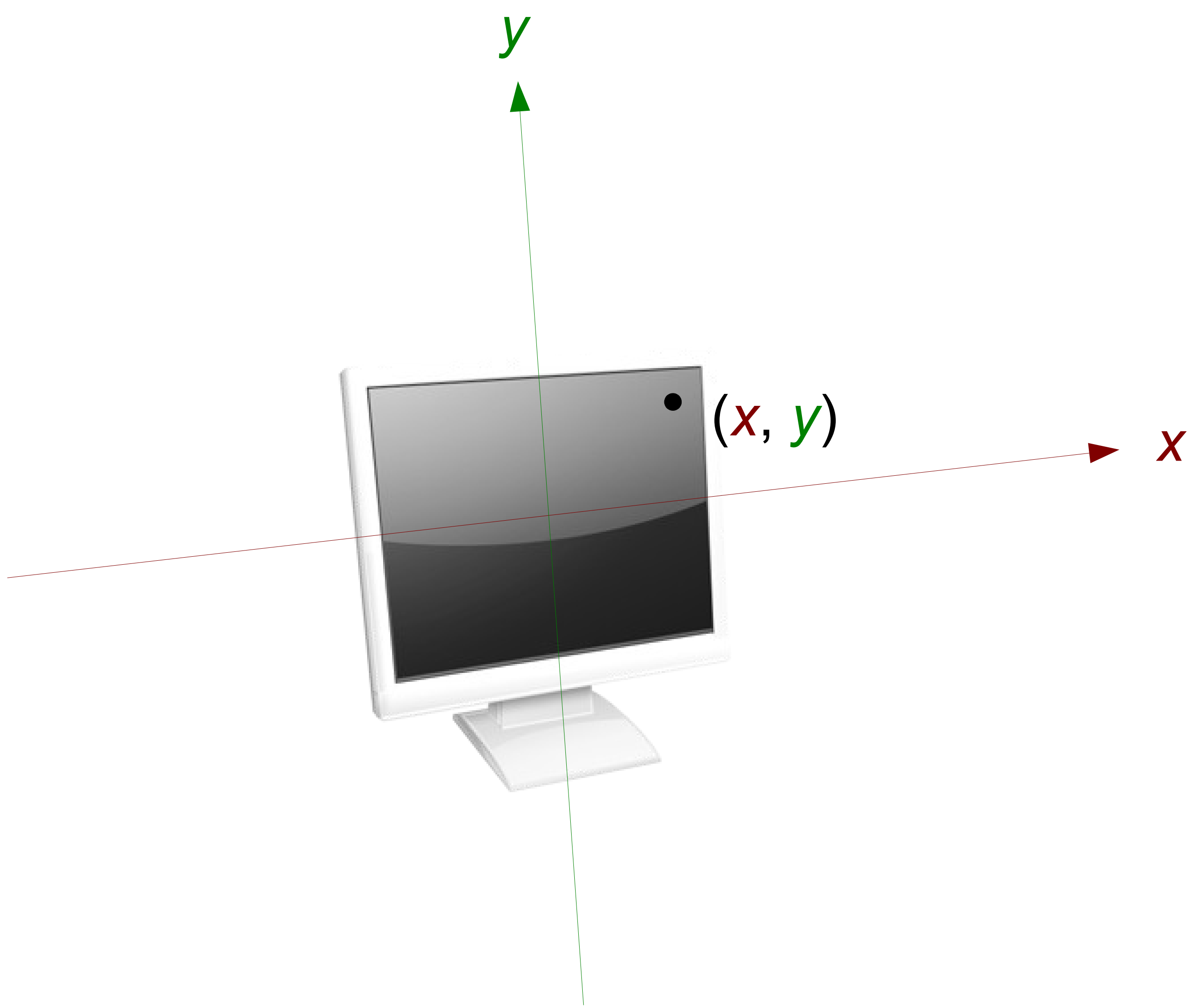


Image credit: DGBthekafu / Wikimedia Commons user Bouncey2k



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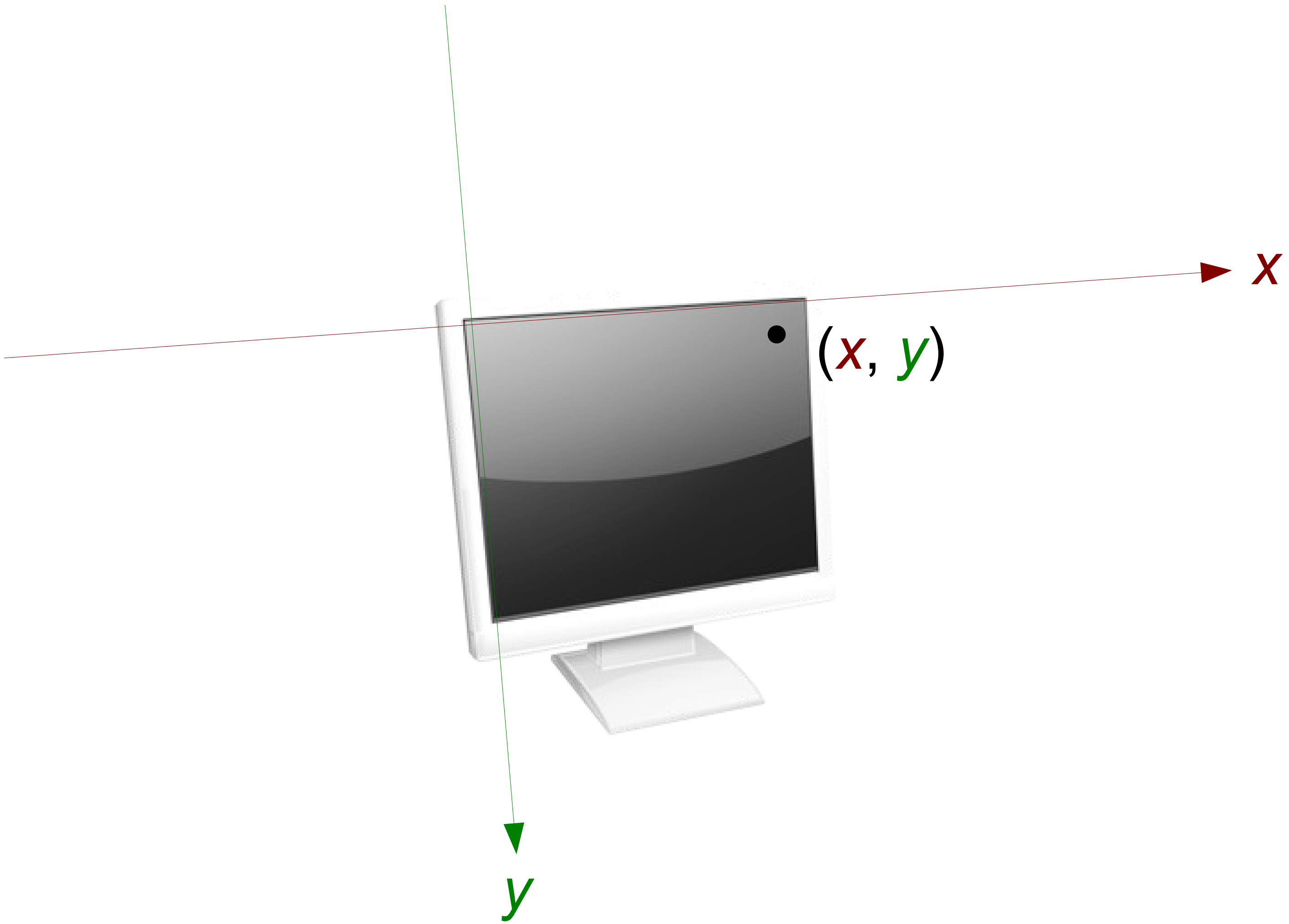
x
(x, y)
y

Pixels

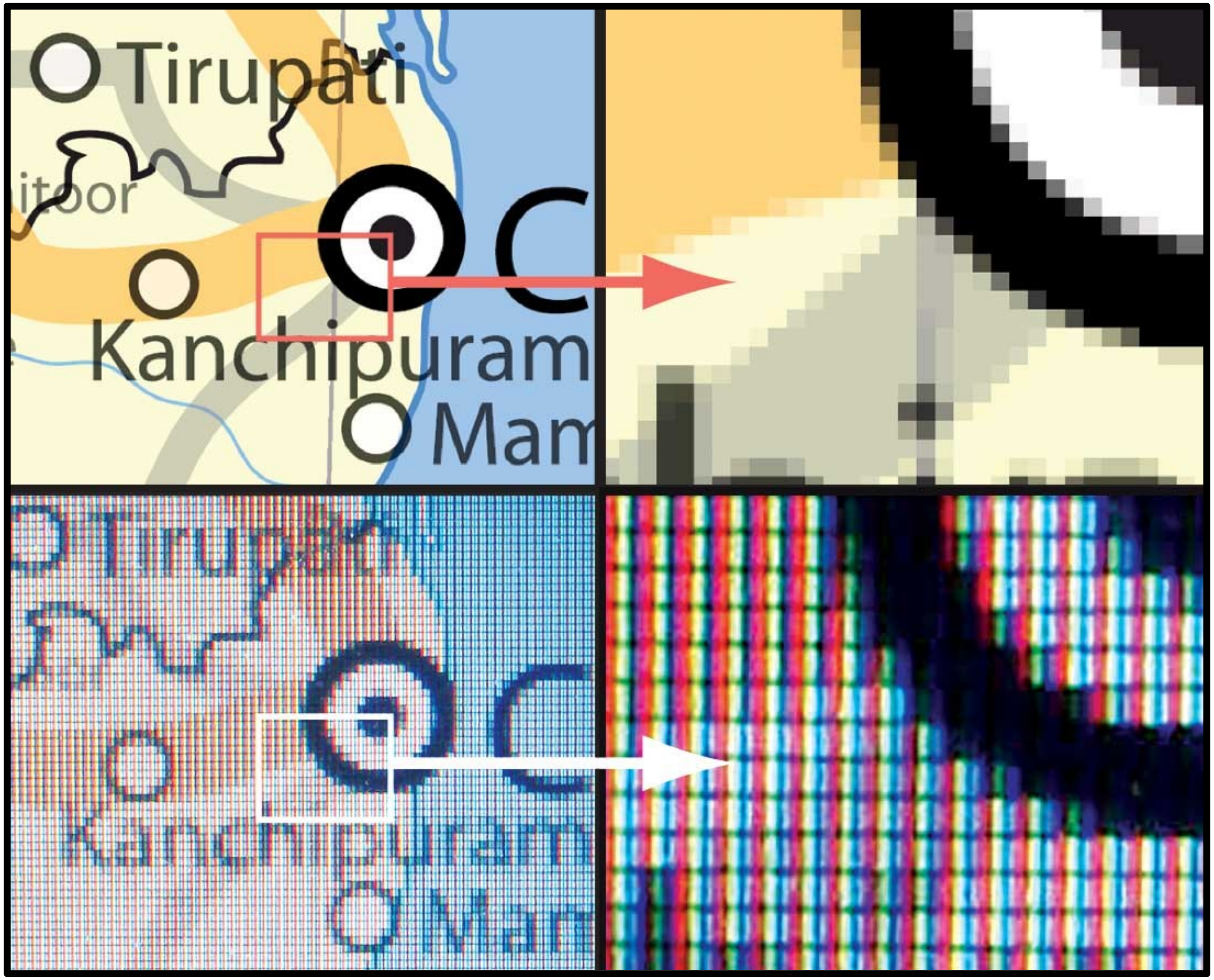

Image credit: Wikimedia Commons user Planemad

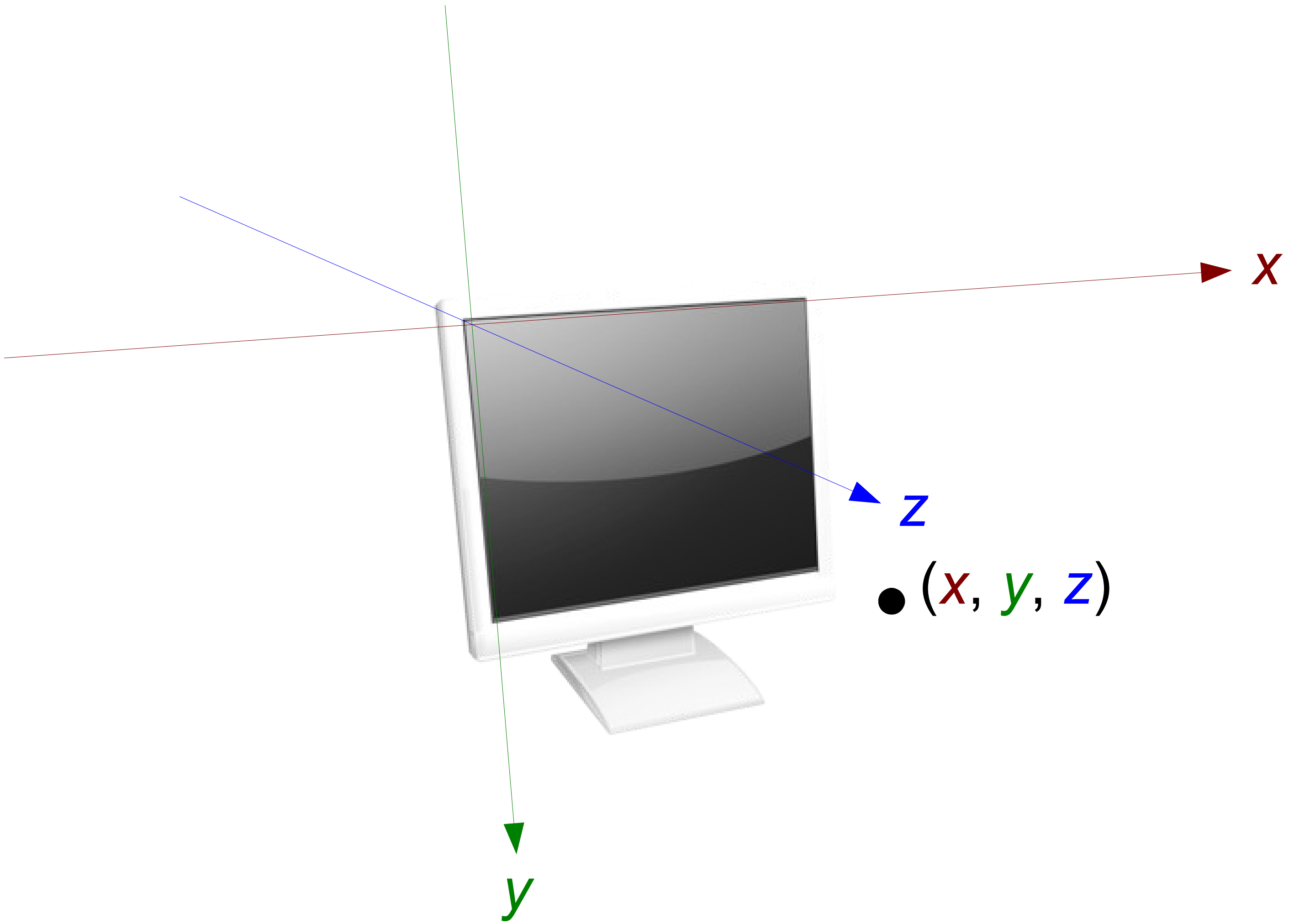

Image credit: DGBthekafu / Wikimedia Commons user Bouncey2k

Demo: Orthographic projection

The pinhole camera

(the *original* graphics hardware)

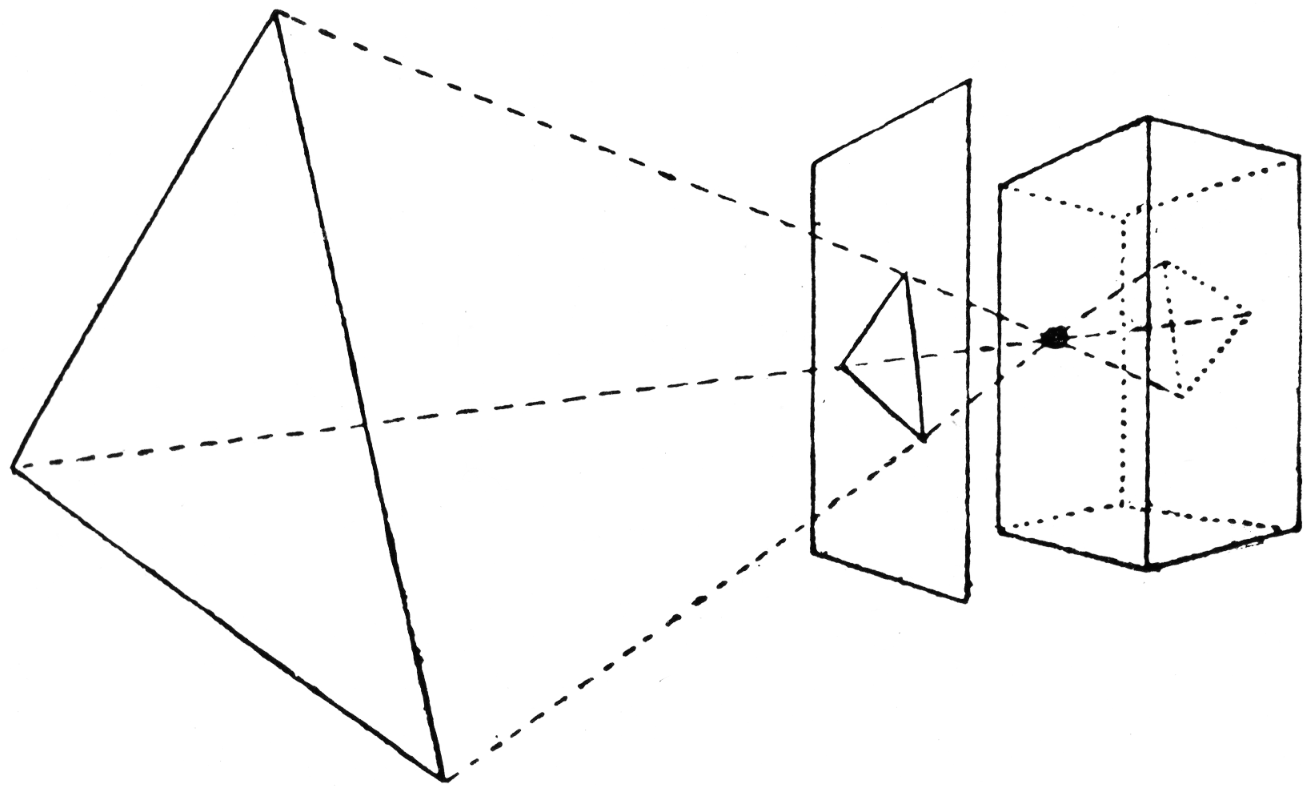

Image credit: L.P. Clerc/Wikimedia Commons user Ateliereleal be

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(the *original* graphics hardware)

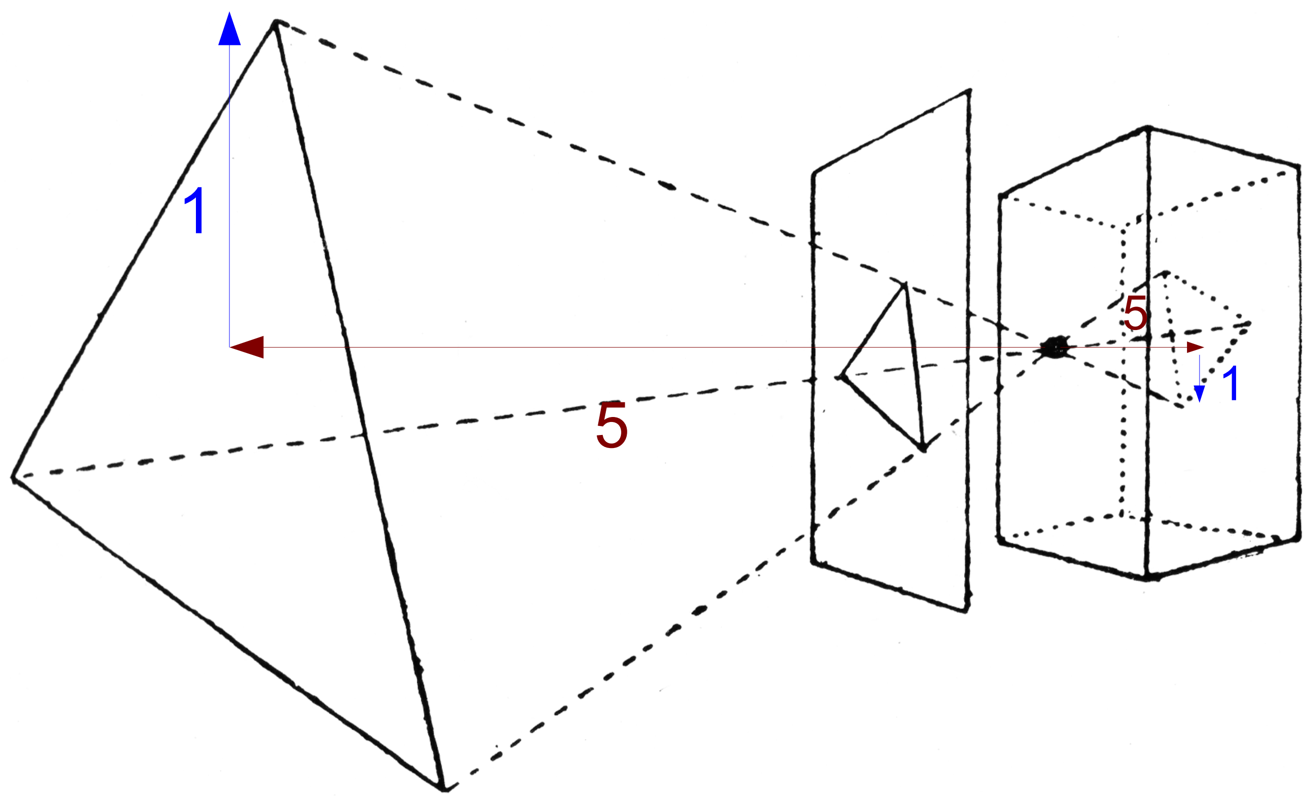

Image credit: L.P. Clerc/Wikimedia Commons user Atelierelealbe

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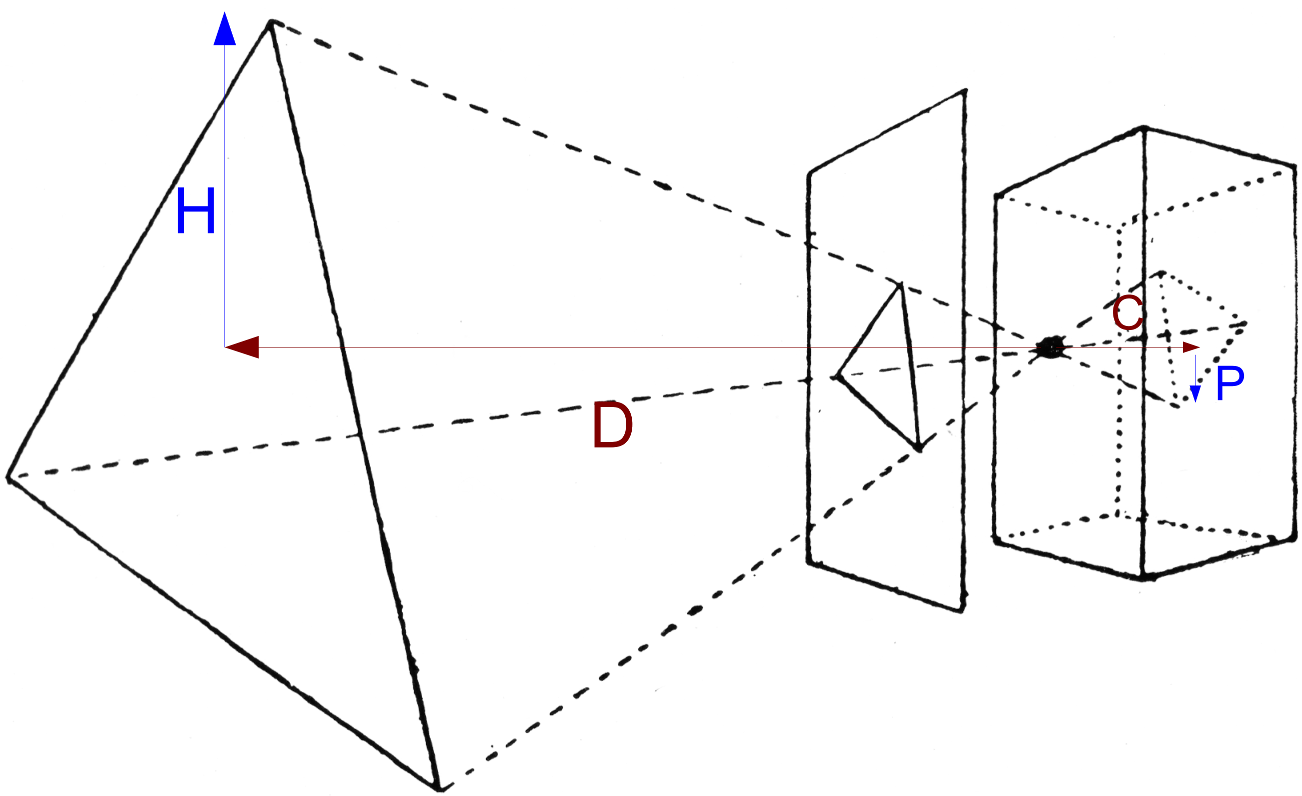
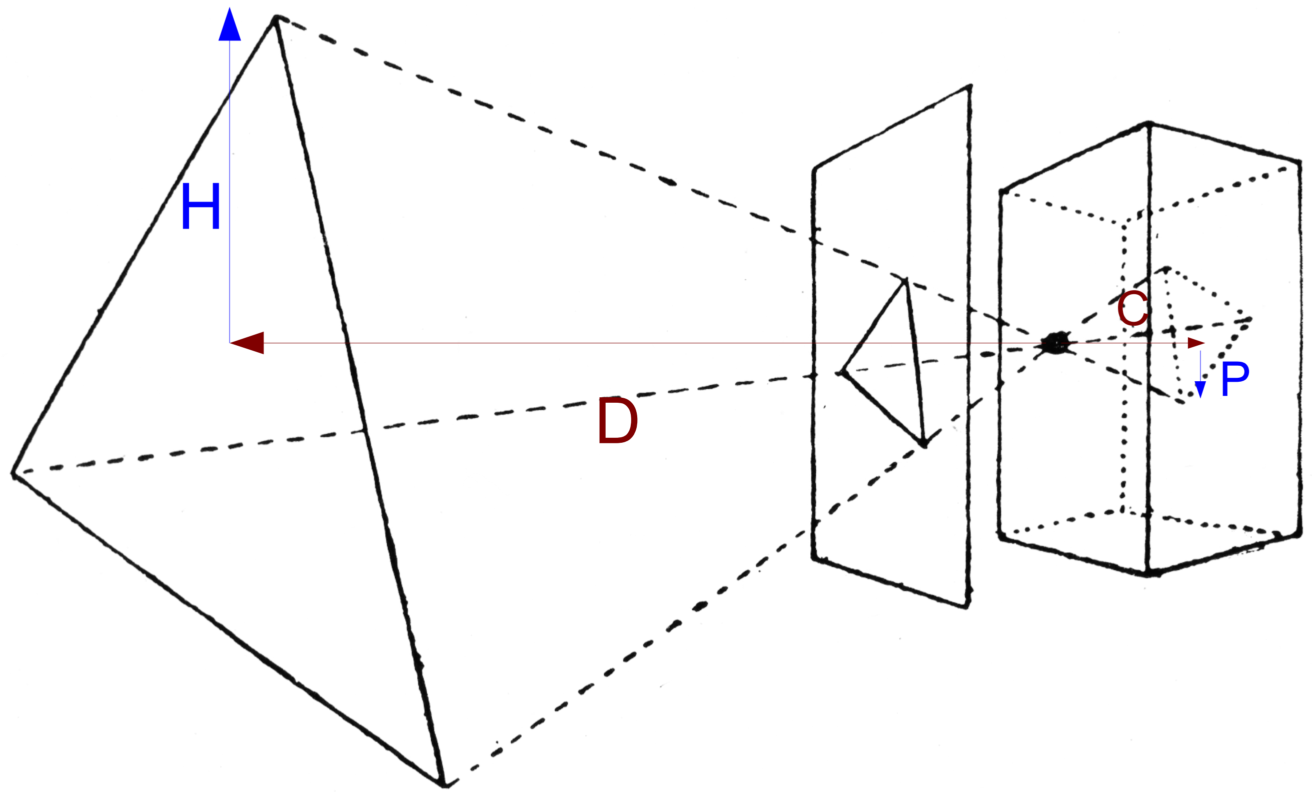


$$\frac{H}{D} = \frac{P}{C}$$

Image credit: L.P. Clerc/Wikimedia Commons user Atelierelealbe

The pinhole camera

(the *original* graphics hardware)



$$\frac{P}{C} = \frac{H}{D}$$

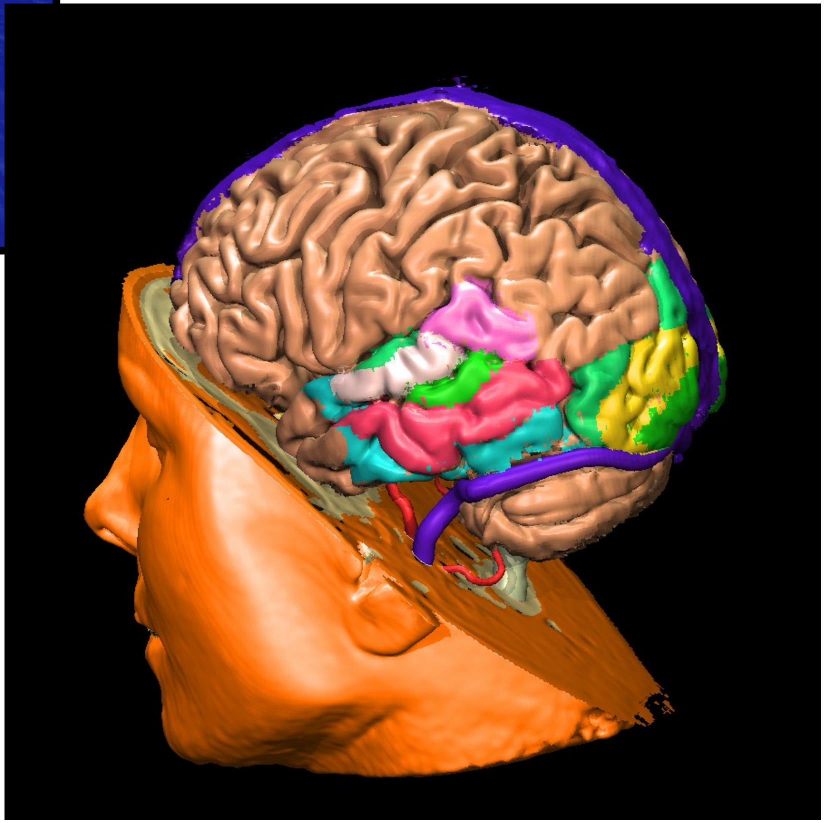
$$P = \frac{HC}{D}$$

Image credit: L.P. Clerc/Wikimedia Commons user Atelierelealbe

Voxels?

Minecraft

Voxel-based
brain imaging



How many voxels
do we need?

Image credits: Mojang AB/Wikimedia Commons users 4urge, Muehlenau

Triangles!

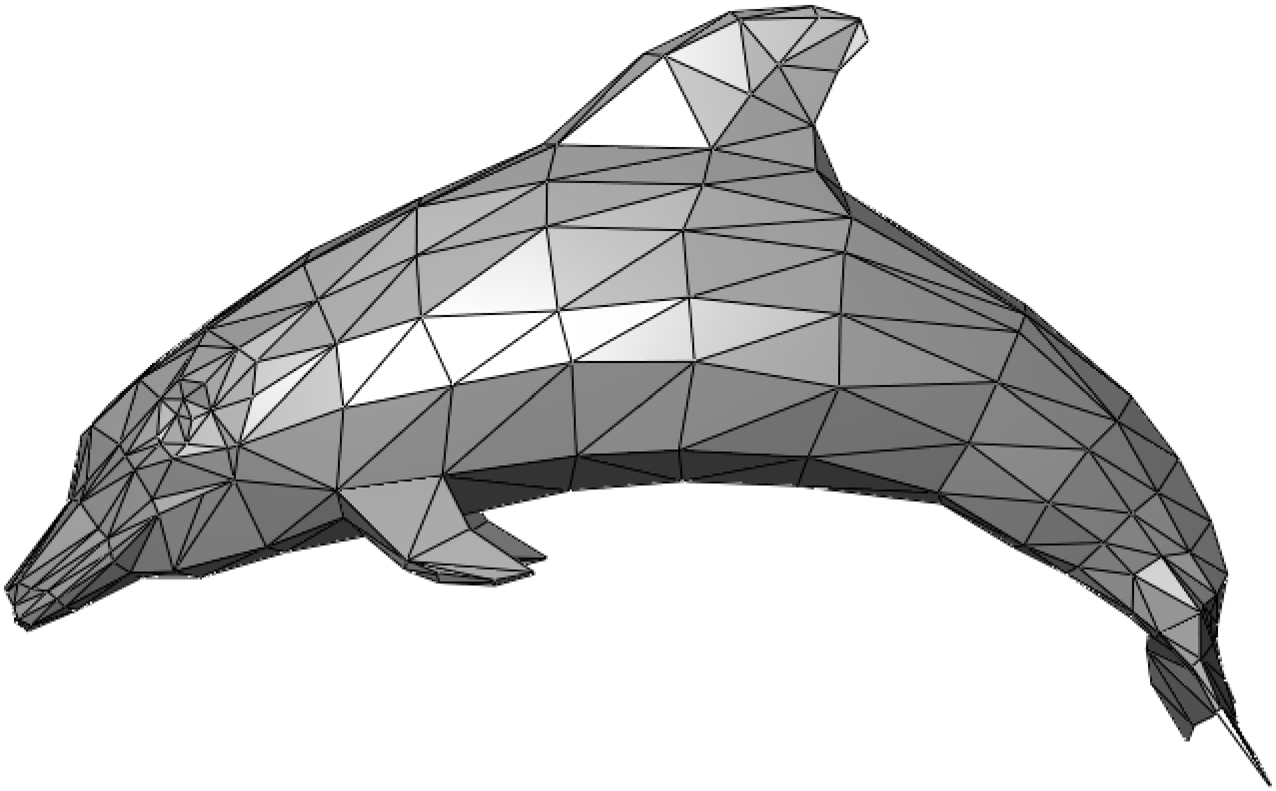


Image credit: Wikimedia Commons user Chrschn

3-D Cartesian coordinates

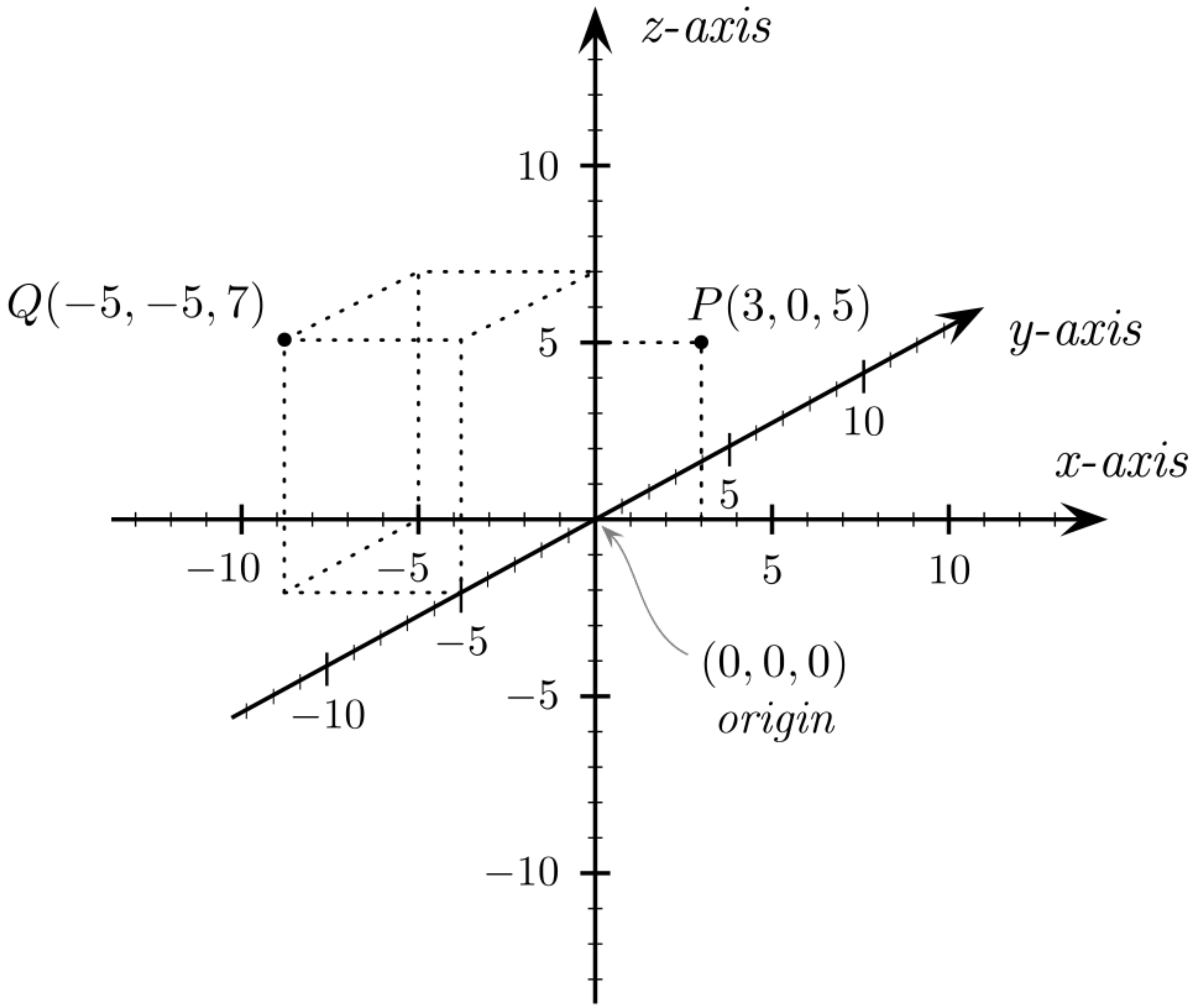


Image credit: Wikimedia Commons user Gustavb

Demo: Journeys of a Teapot

Vector addition

$$a = (5, 6, -3)$$
$$b = (-1, 7, 2)$$

$$a + b = (5 + (-1), 6 + 7, (-3) + 2)$$
$$= (4, 13, -1)$$

Vector addition

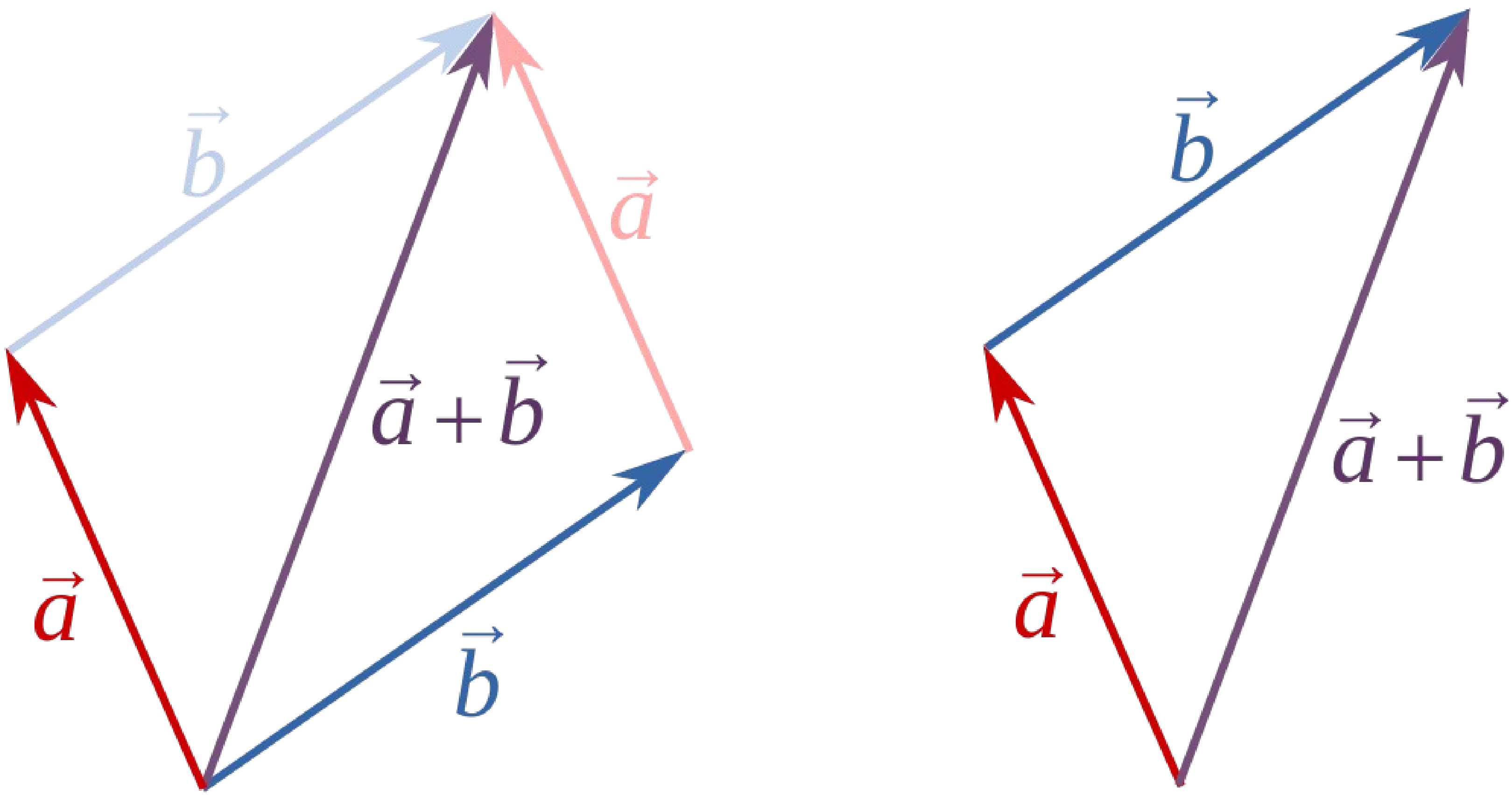


Image credit: Wikimedia Commons user 石庭豐

Points → vectors from the origin

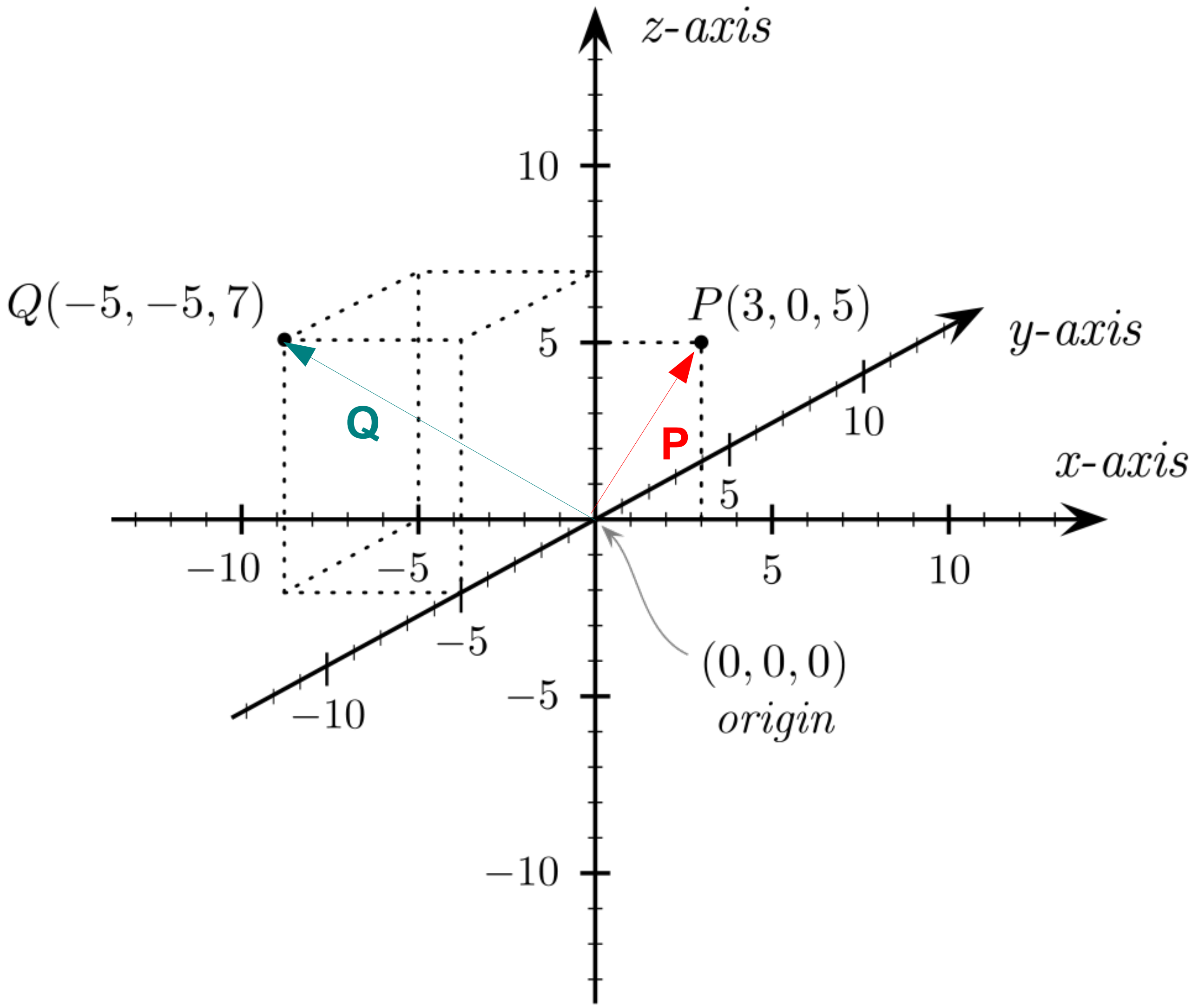


Image credit: Wikimedia Commons user Gustavb

Relative positioning of objects

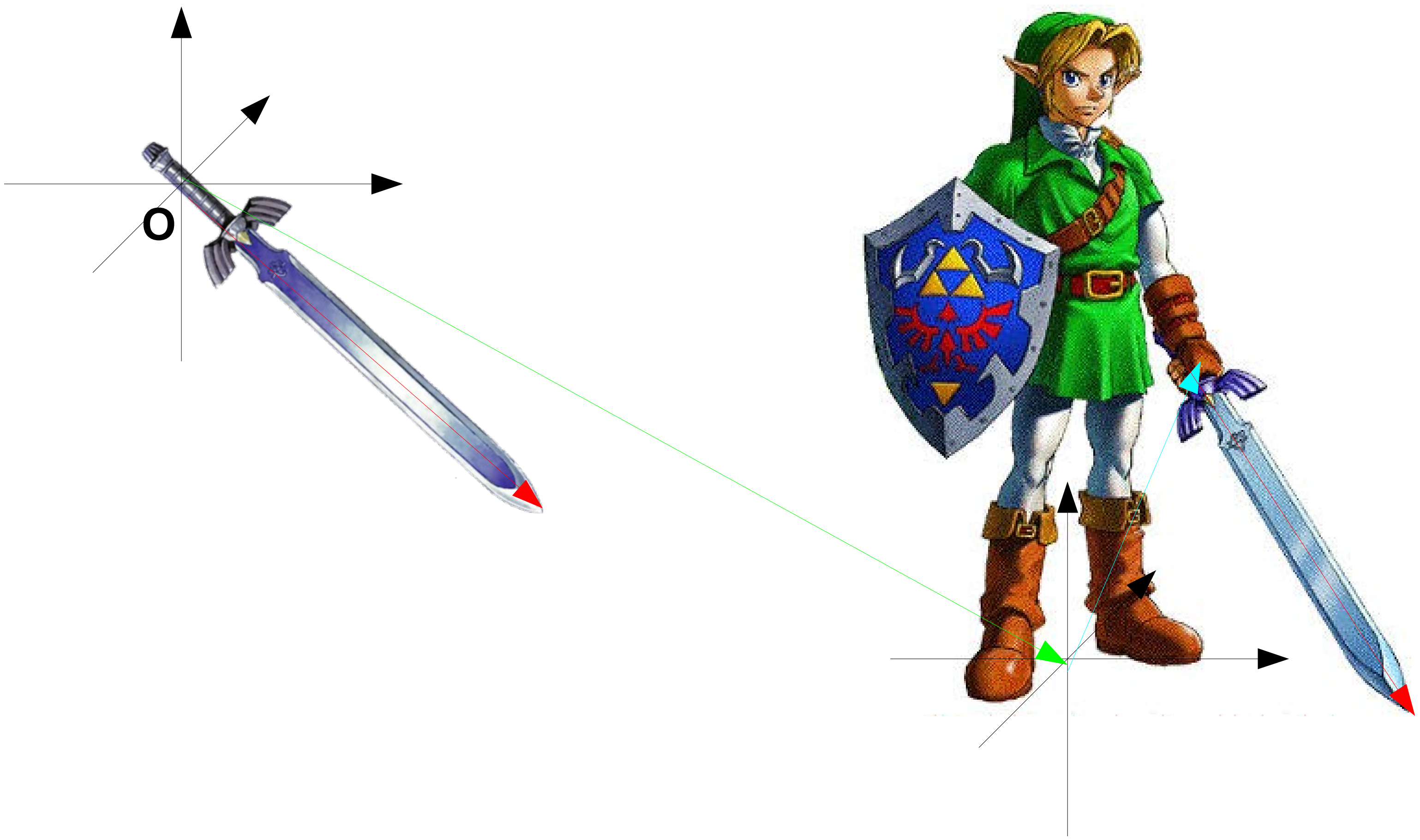


Image credits: kasuto.net, zelda.wikia.com

Demo: Placing the camera

Vector subtraction

$a = (5, 6, -3)$
$b = (-1, 7, 2)$

$$a - b = (5 - (-1), 6 - 7, (-3) - 2)$$
$$= (6, -1, -5)$$

Vector subtraction

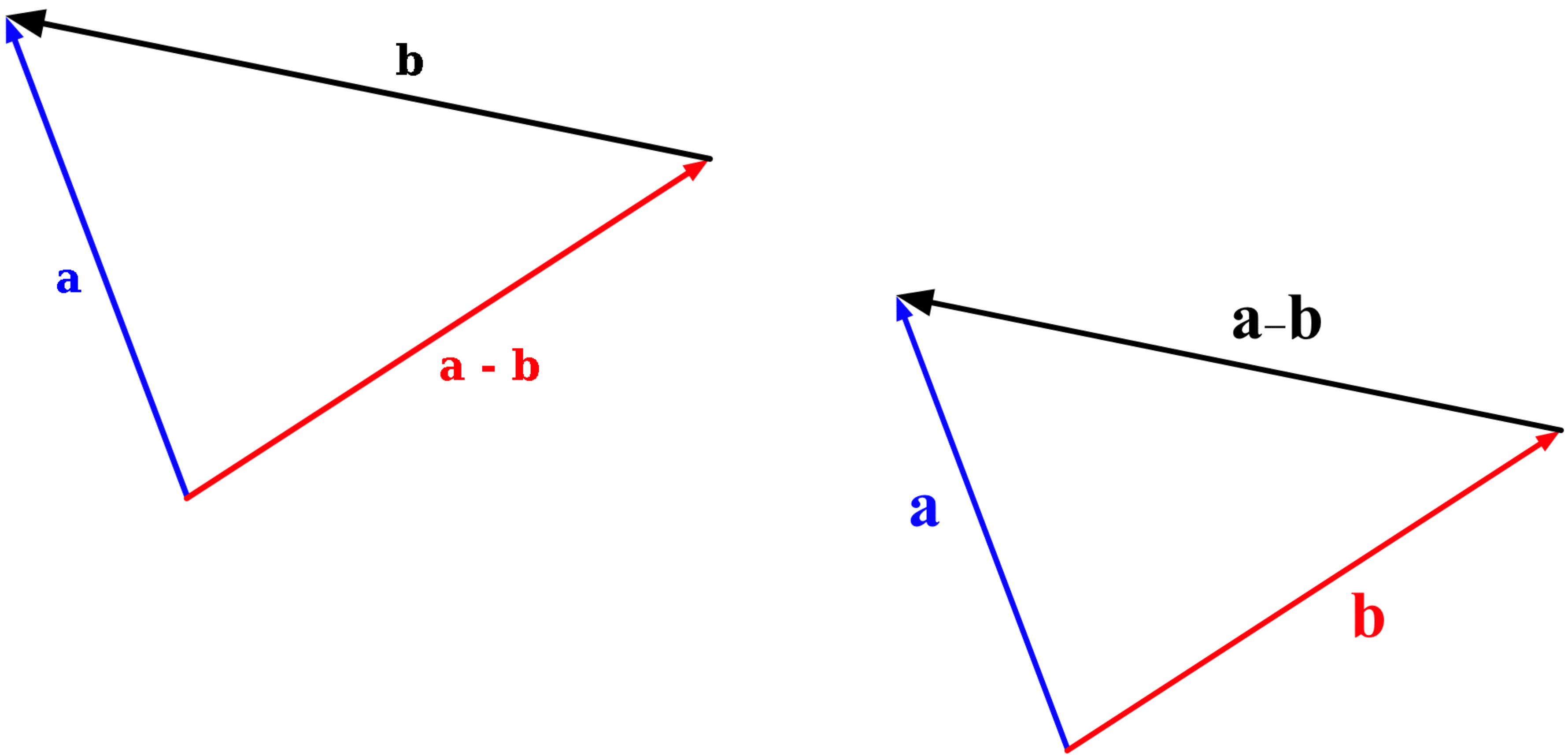


Image credit: Wikimedia Commons user bdesham

Scalar multiplication

$$a = (5, 6, -3)$$

$$5a = (5 \cdot 5, 5 \cdot 6, 5 \cdot (-3))$$
$$= (25, 30, -15)$$

Scalar multiplication

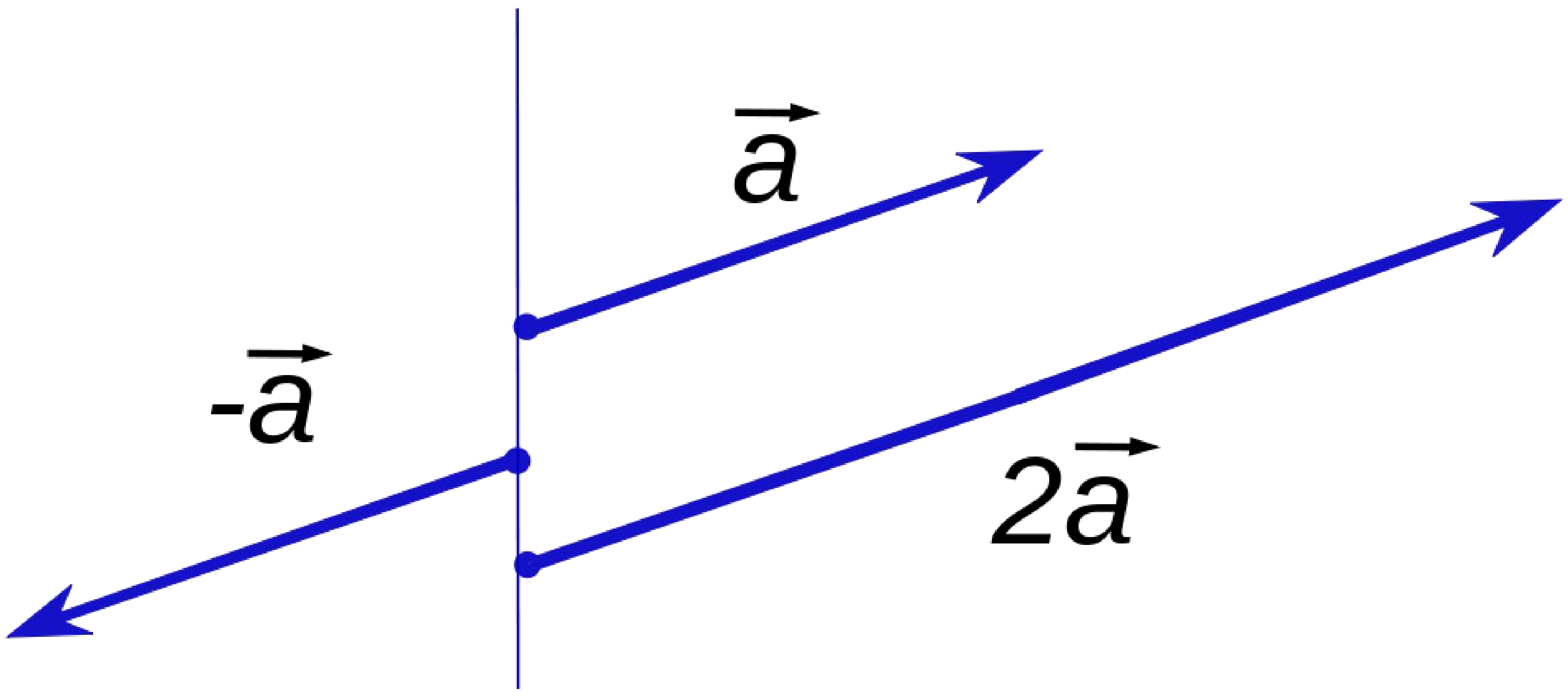


Image credit: Silly rabbit/Wikimedia Commons user Konradek

Demo: Placing the camera

Moving objects

Image credit: Michael Maggs/Richard Bartz

Applying speed in small steps

(This is called "Euler's method for numerical integration."
No, you don't have to remember that. But you can if you want.)

v = 5 m/s

Image credit: Wikimedia Commons user MrX

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v = 5 m/s

t = 1/60 s

Image credit: Wikimedia Commons user MrX

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v = 5 m/s

t = 1/60 s

$d = v \cdot t$ = 1/12 m

Image credit: Wikimedia Commons user MrX

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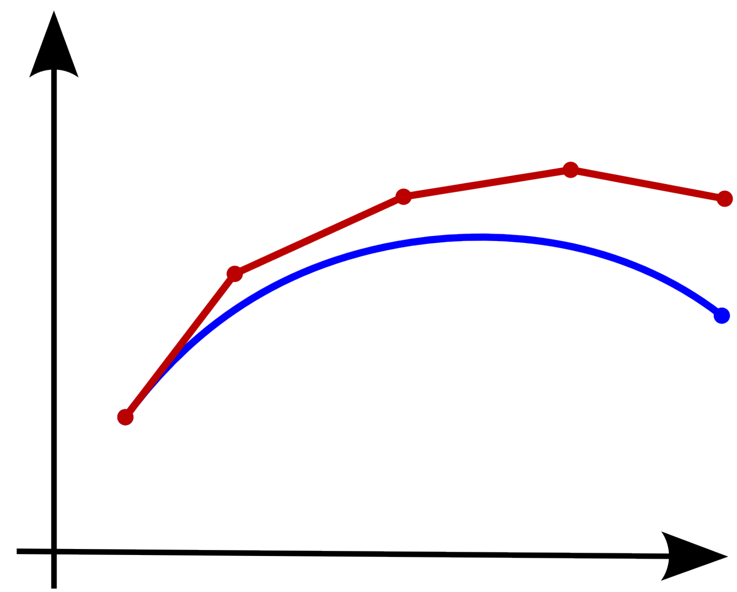


Image credit: Wikimedia Commons user MrX

Demo: Moving the teapot

The dot product

$$a = (5, 6, -3)$$
$$b = (-1, 7, 2)$$

$$a \cdot b = 5 \cdot (-1) + 6 \cdot 7 + (-3) \cdot 2$$
$$= -5 + 42 + -6 = 31$$

An illustrative example

	Heller (R)	Reid (D)
37	Y	Y
38	Y	N
39	N	Y
40	N	N
41	N	Y
42	Y	N
43	Y	N
45	N	N
46	Y	Y
54	N	Y

Source: http://govtrack.us/

An illustrative example

	Heller (R)	Reid (D)	product
37	**+1**	**+1**	+1
38	**+1**	**-1**	-1
39	**-1**	**+1**	-1
40	**-1**	**-1**	+1
41	**-1**	**+1**	-1
42	**+1**	**-1**	-1
43	**+1**	**-1**	-1
45	**-1**	**-1**	+1
46	**+1**	**+1**	+1
54	**-1**	**+1**	-1

Total:
-2

Source: http://govtrack.us/

An illustrative example

	Boxer (D)	Feinstein (D)
37	N	N
38	N	N
39	Y	Y
40	N	N
41	Y	Y
42	N	N
43	N	N
45	N	N
46	Y	Y
54	Y	Y

Source: http://govtrack.us/

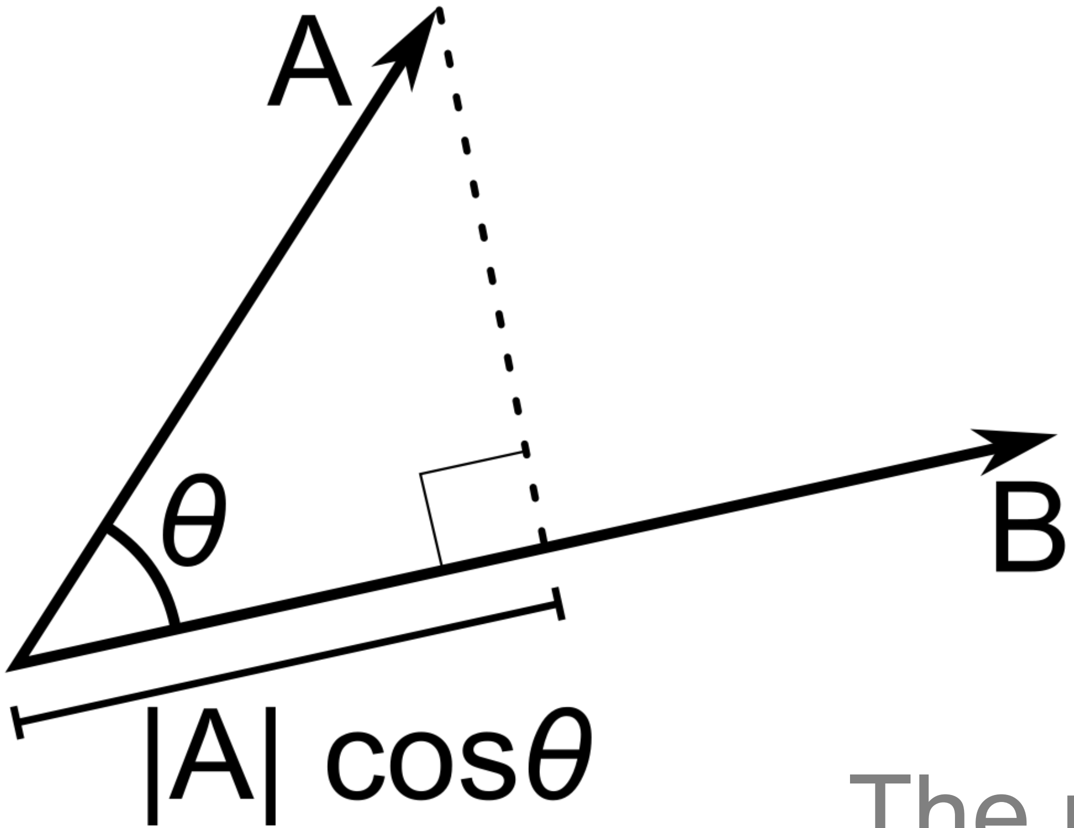
An illustrative example

	Boxer (D)	Feinstein (D)	product
37	-1	-1	+1
38	-1	-1	+1
39	+1	+1	+1
40	-1	-1	+1
41	+1	+1	+1
42	-1	-1	+1
43	-1	-1	+1
45	-1	-1	+1
46	+1	+1	+1
54	+1	+1	+1

Total:
+10 (!)

Source: http://govtrack.us/

Projecting one vector onto another



The real dot product:

|A| **|B|** cos θ

Image credit: Wikimedia Commons user Mazin07

Lighting matters

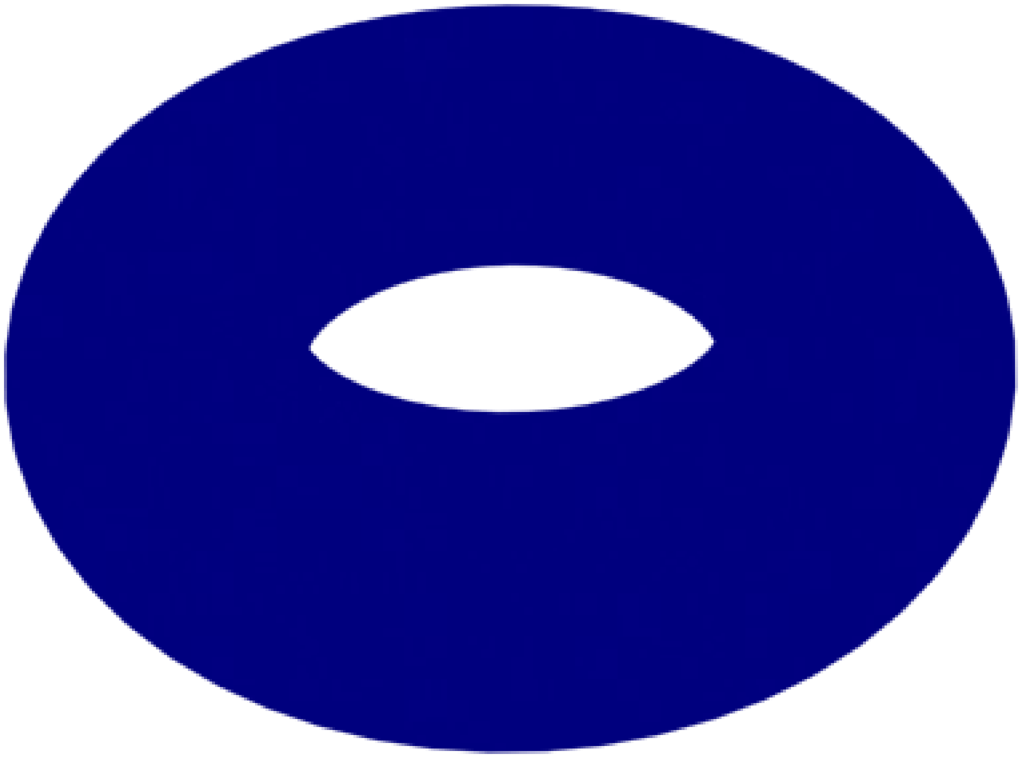


Image credit: Wikimedia Commons user Pokipsy76

Lighting matters

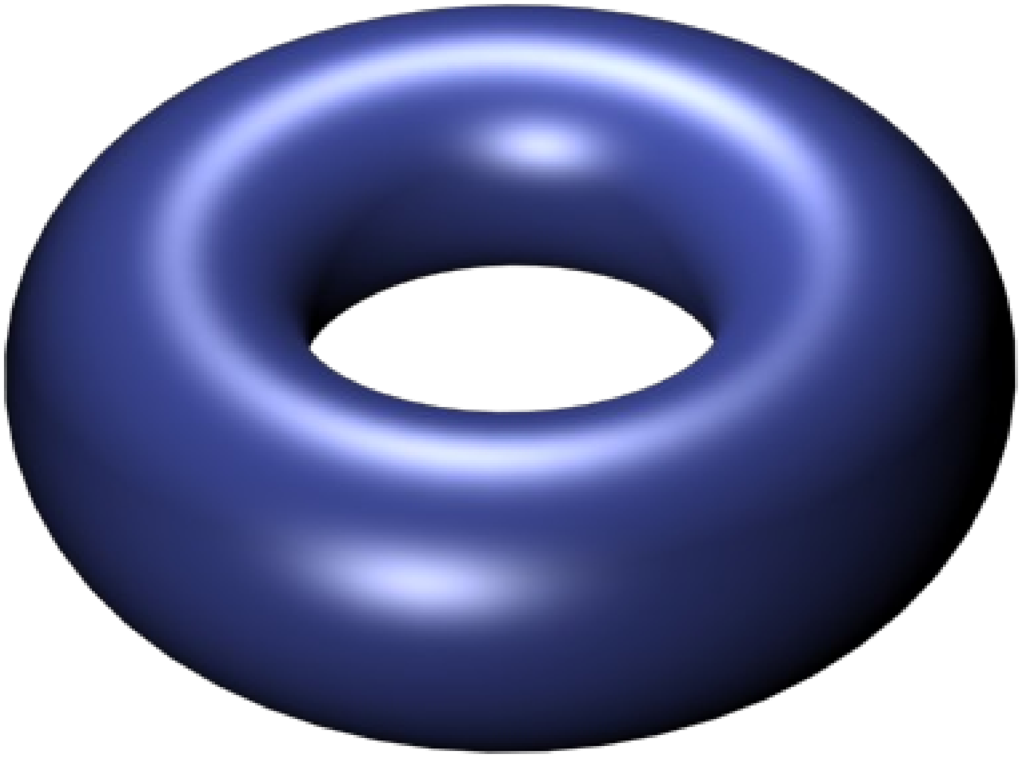
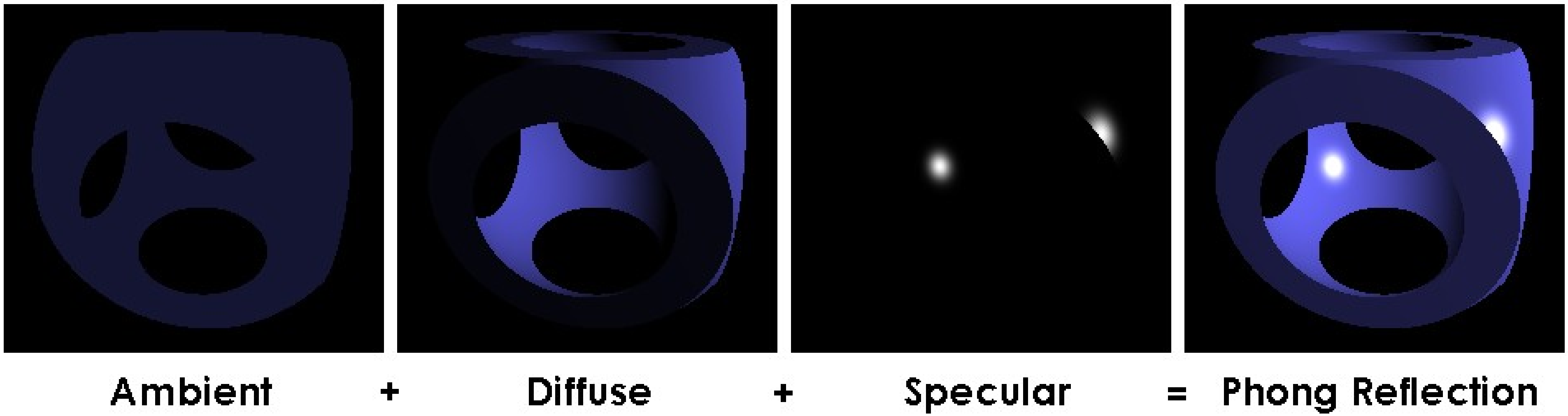


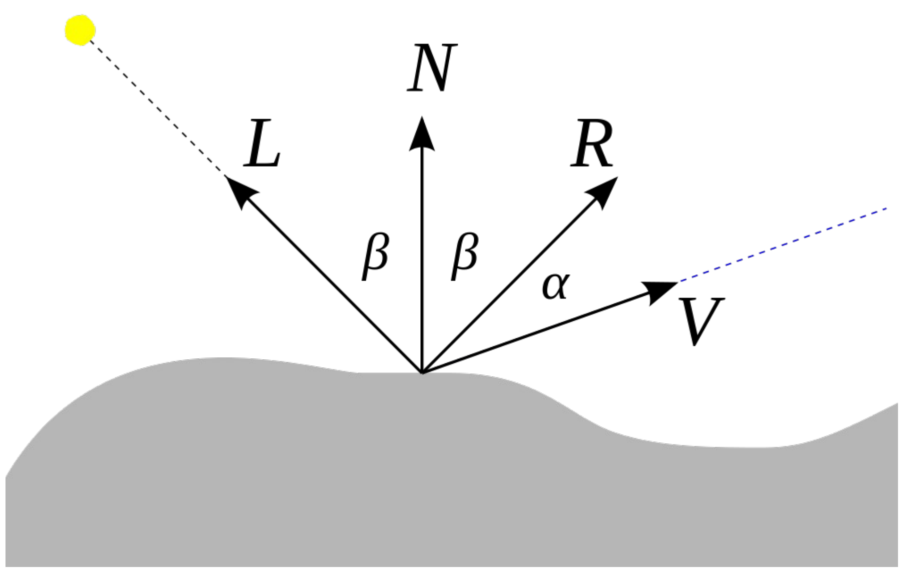
Image credit: Wikimedia Commons user Pokipsy76

Video: Phong shading

Lighting

The Phong illumination model





Ambient: constant

Diffuse: $\mathbf{L} \cdot \mathbf{N} = \cos\beta$

Specular: $(\mathbf{R} \cdot \mathbf{V})^k = (\cos\alpha)^k$

Image credit: Brad Smith/Wikimedia commons user Rainwarrior

Demo: Turning on the sun

The cross product

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \wedge \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} \quad \\ \quad \\ \quad \end{pmatrix}$$

Image credit: Wikimedia Commons user Ravahimanana

The cross product

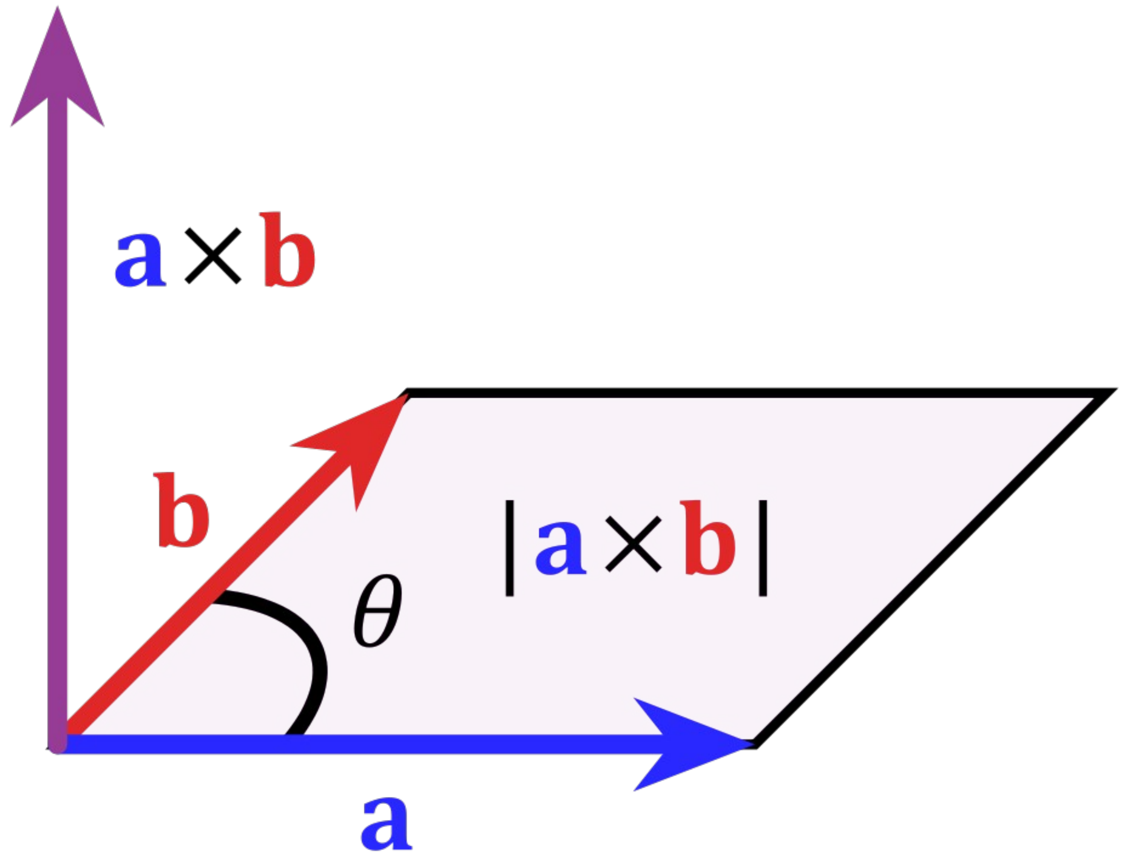


Image credit: Wikimedia Commons user Acdx

Thank you!

Code and slides:
http://stanford.edu/~wmonroe4/splash

(wait a day or two)