VECTORS WITH VIDEO GAMES

Will Monroe
Splash! Teaching Program
April 22, 2012

Image credit: Wikimedia Commons user Manop

Video: Portal clip

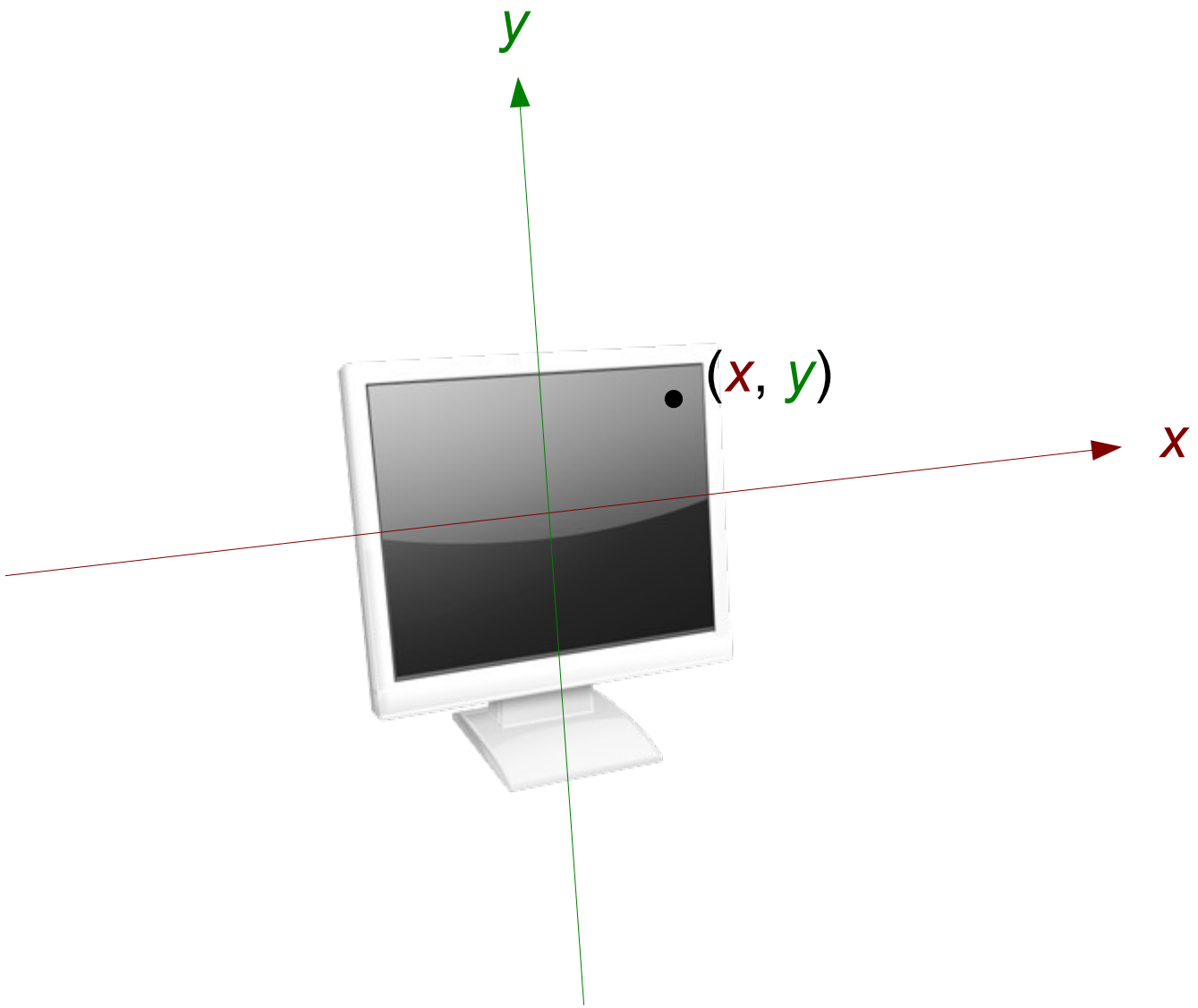
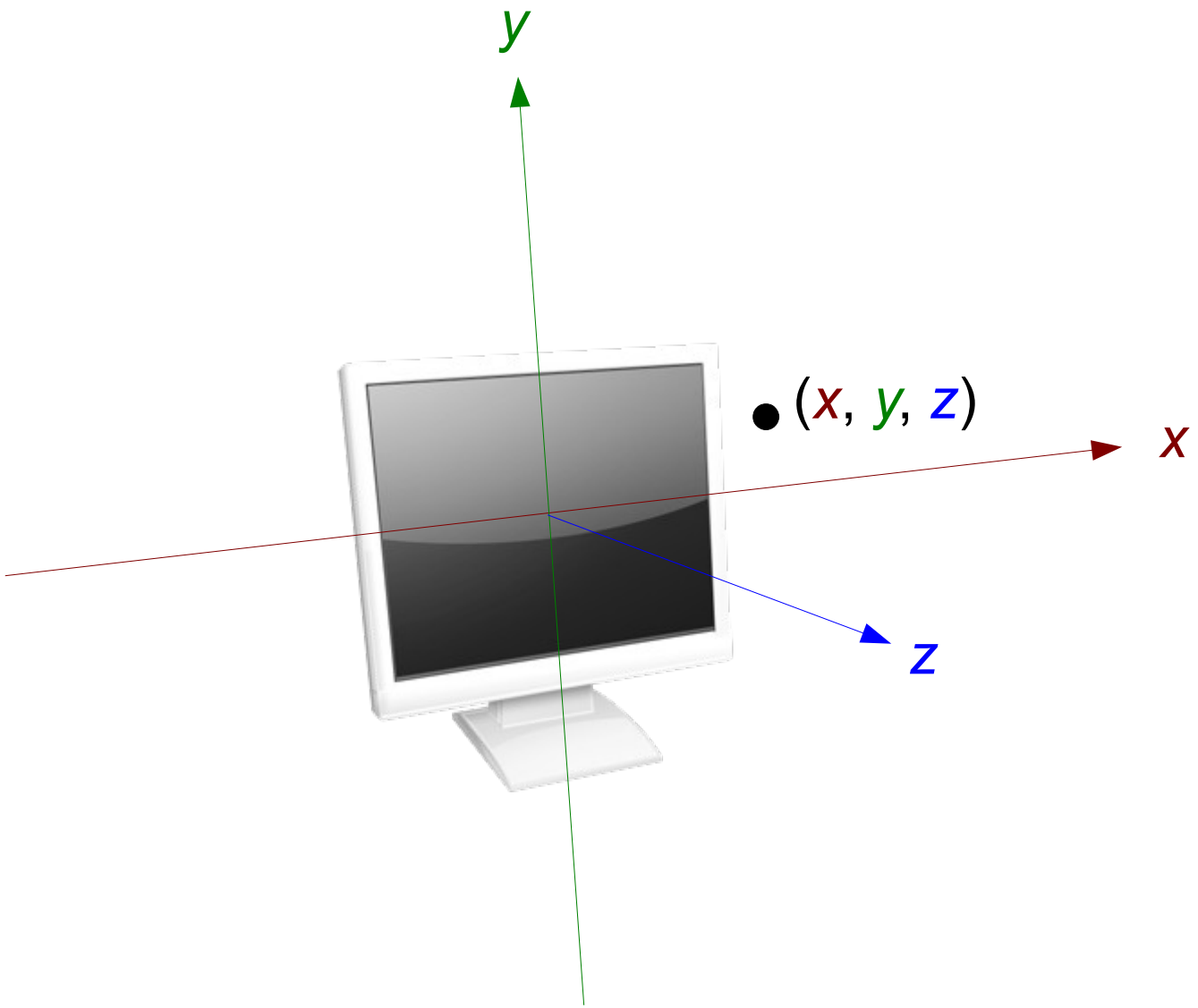


Image credit: DGBthekafu / Wikimedia Commons user Bouncey2k


y
(x, y, z)
x
z

The pinhole camera

(the *original* graphics hardware)

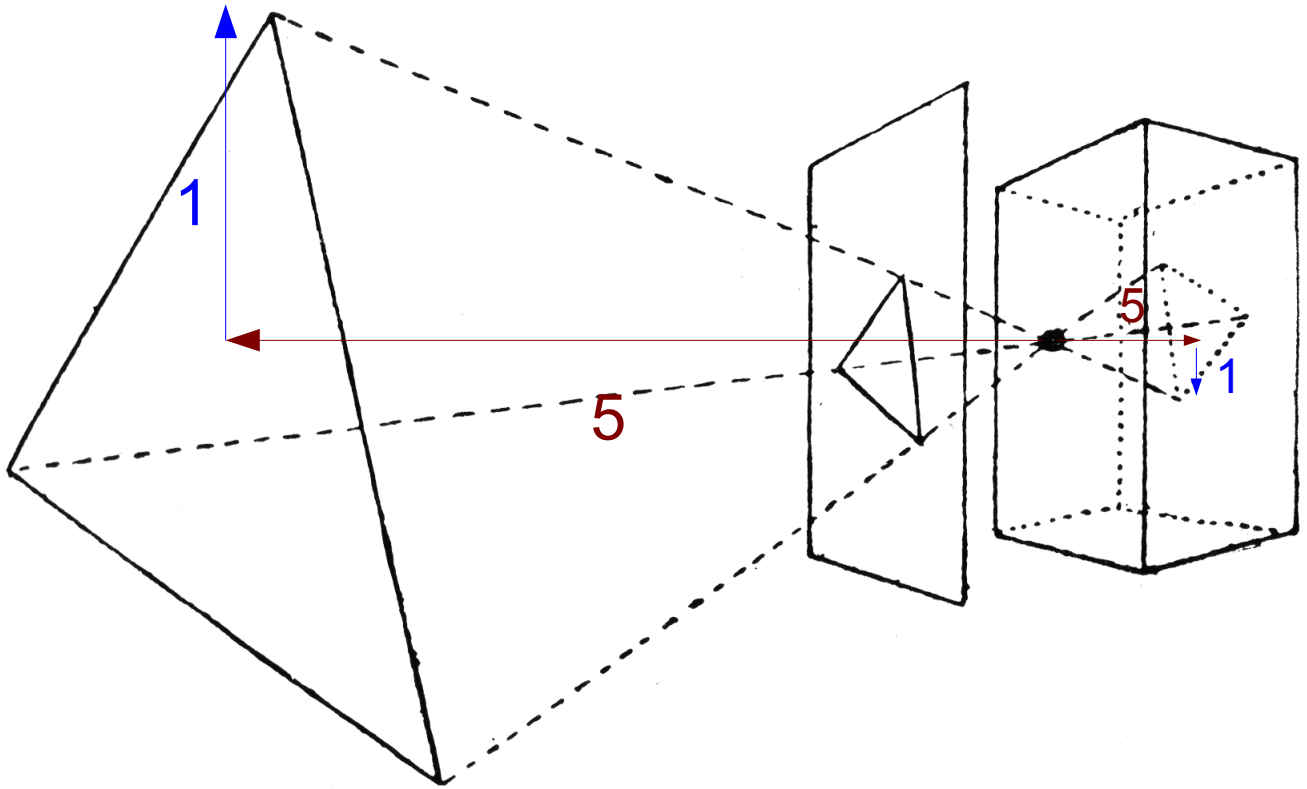


Image credit: L.P. Clerc/Wikimedia Commons user Atelierelealbe

Pixels

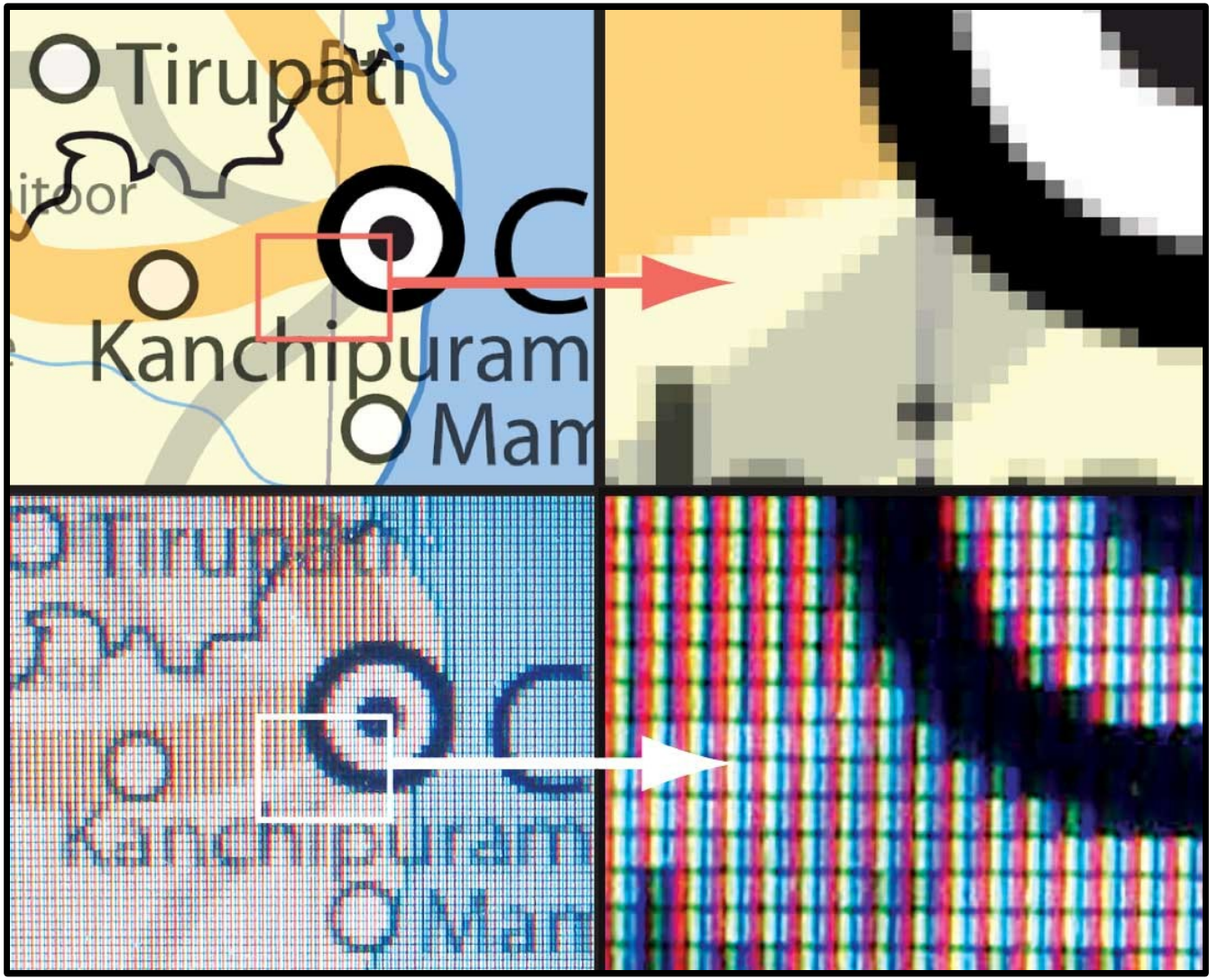
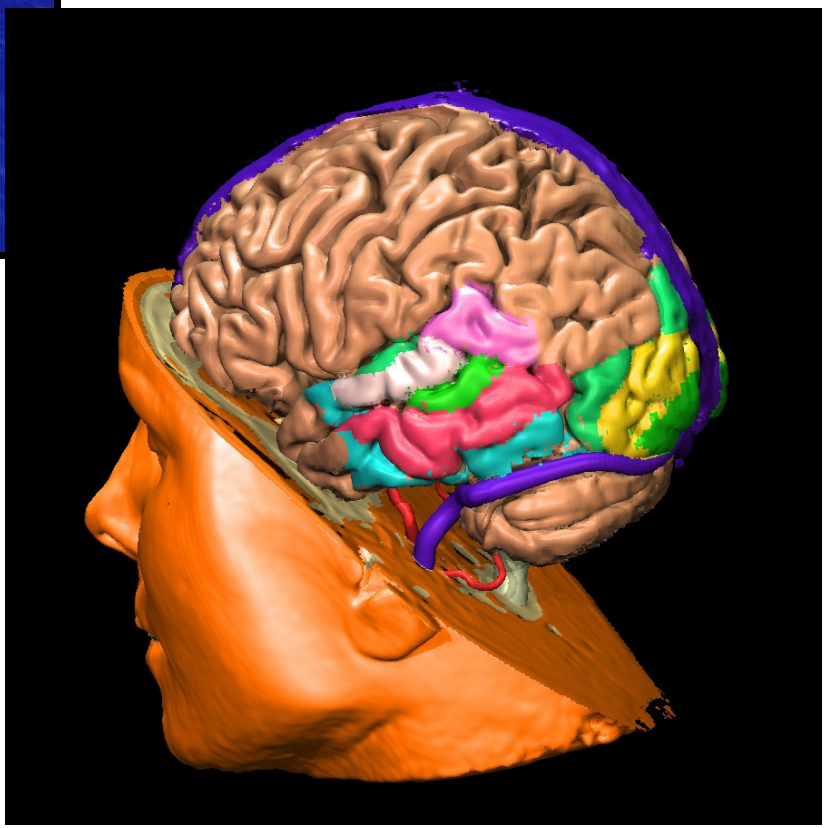


Image credit: Wikimedia Commons user Planemad

Voxels?

Minecraft

Voxel-based
brain imaging



How many voxels
do we need?

Image credits: Mojang AB/Wikimedia Commons users 4urge, Muehlenau

Triangles!

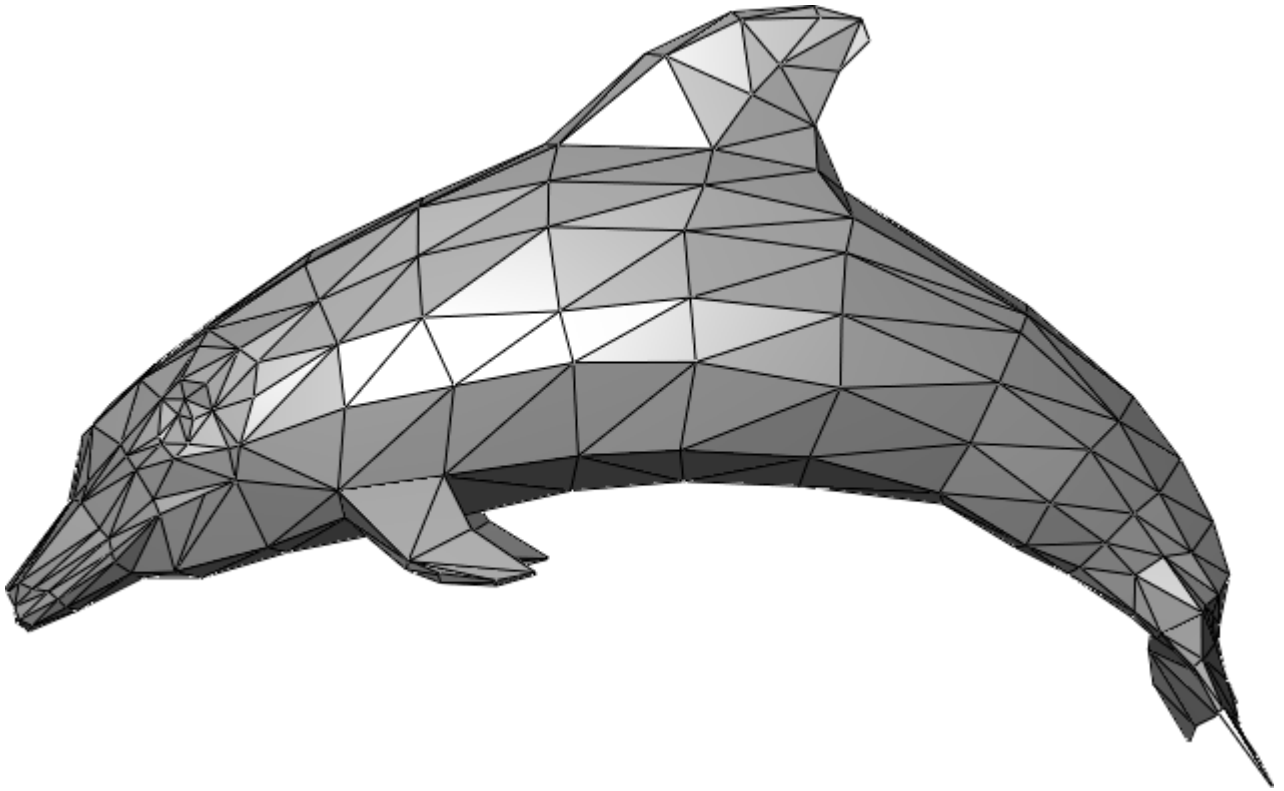


Image credit: Wikimedia Commons user Chrschn

3-D Cartesian coordinates

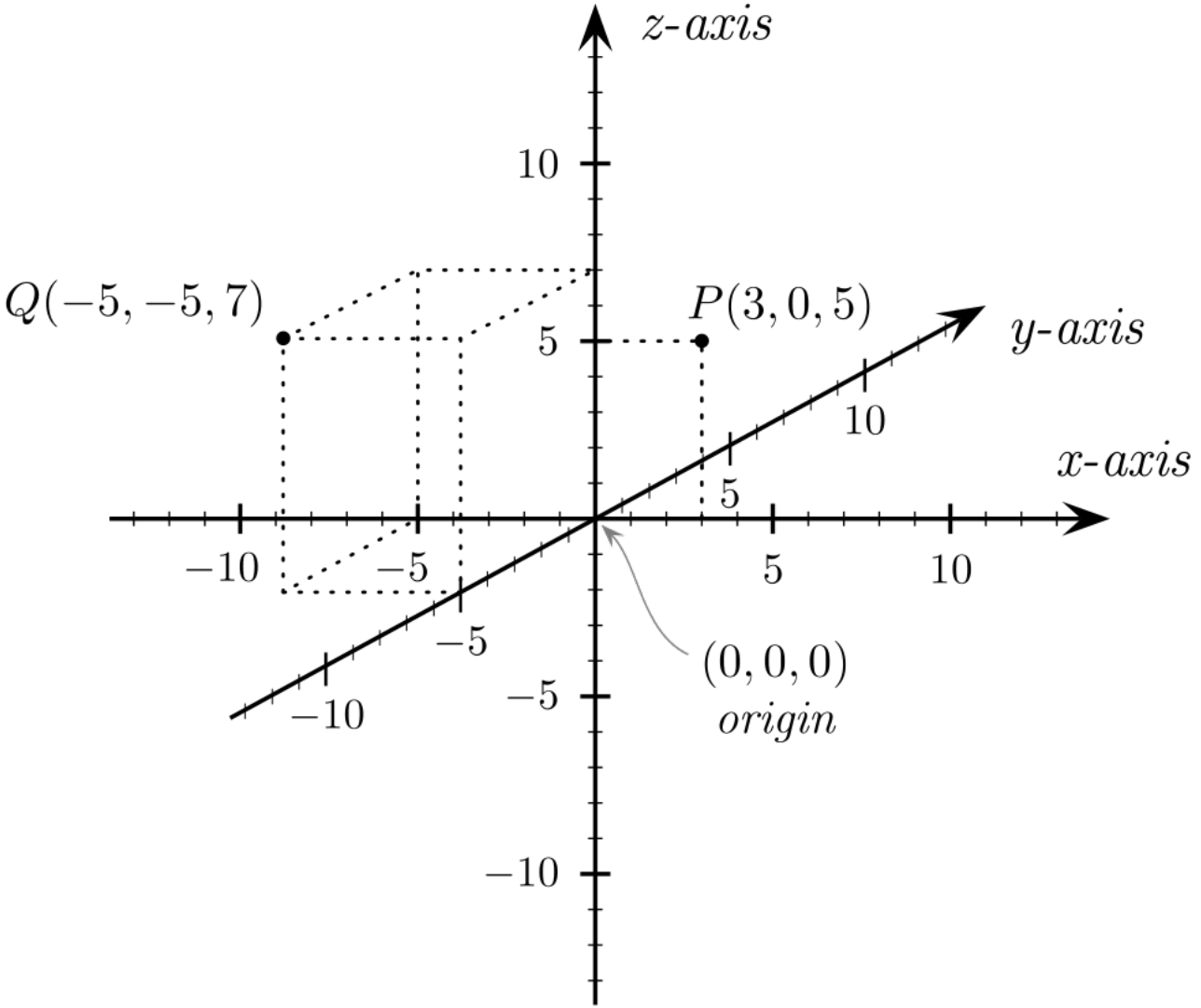


Image credit: Wikimedia Commons user Gustavb

Vector addition

$a = (5, 6, -3)$

$b = (-1, 7, 2)$

$$a + b = (5 + (-1), 6 + 7, (-3) + 2)$$
$$= (4, 13, -1)$$

Vector addition

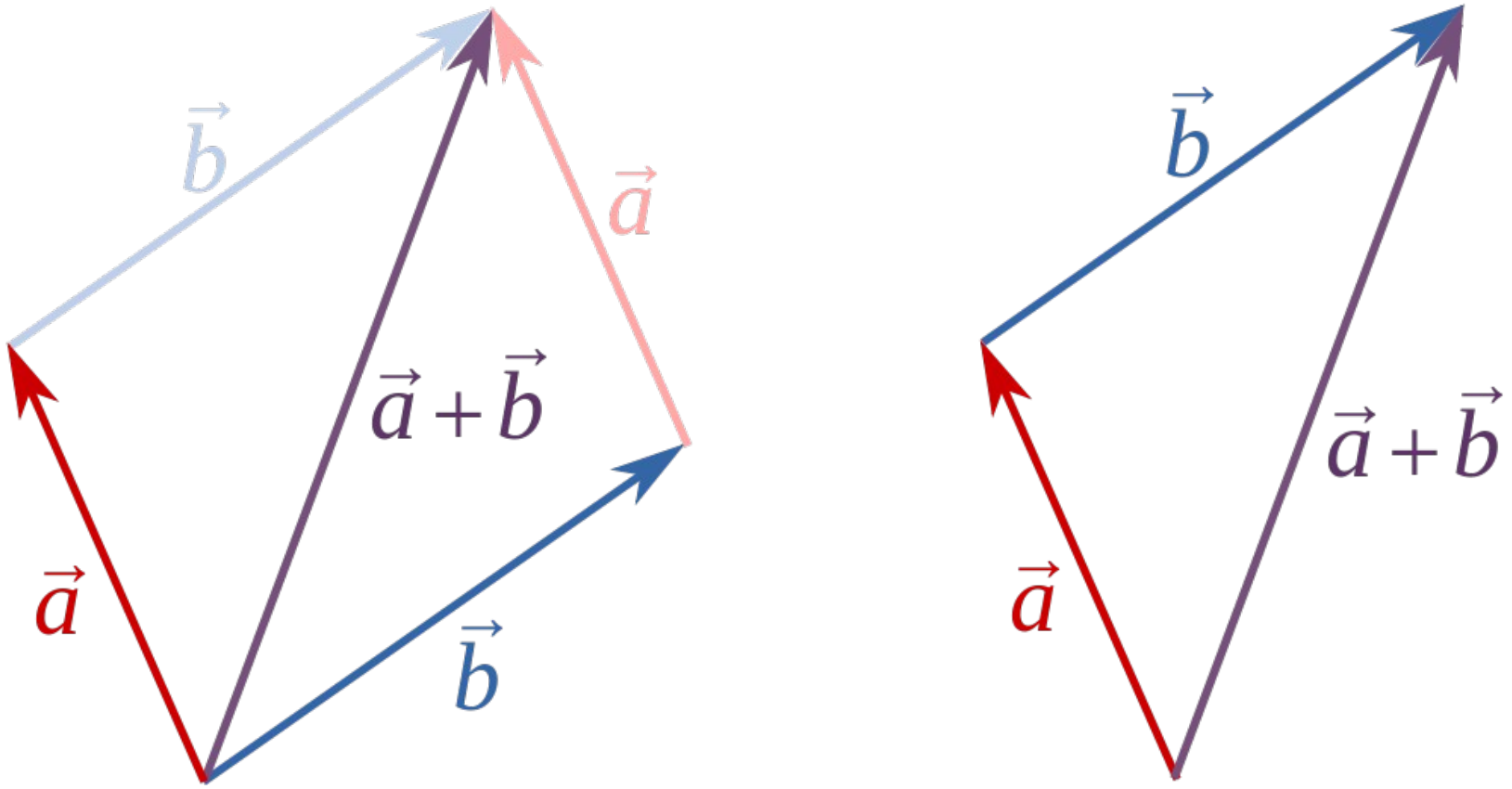
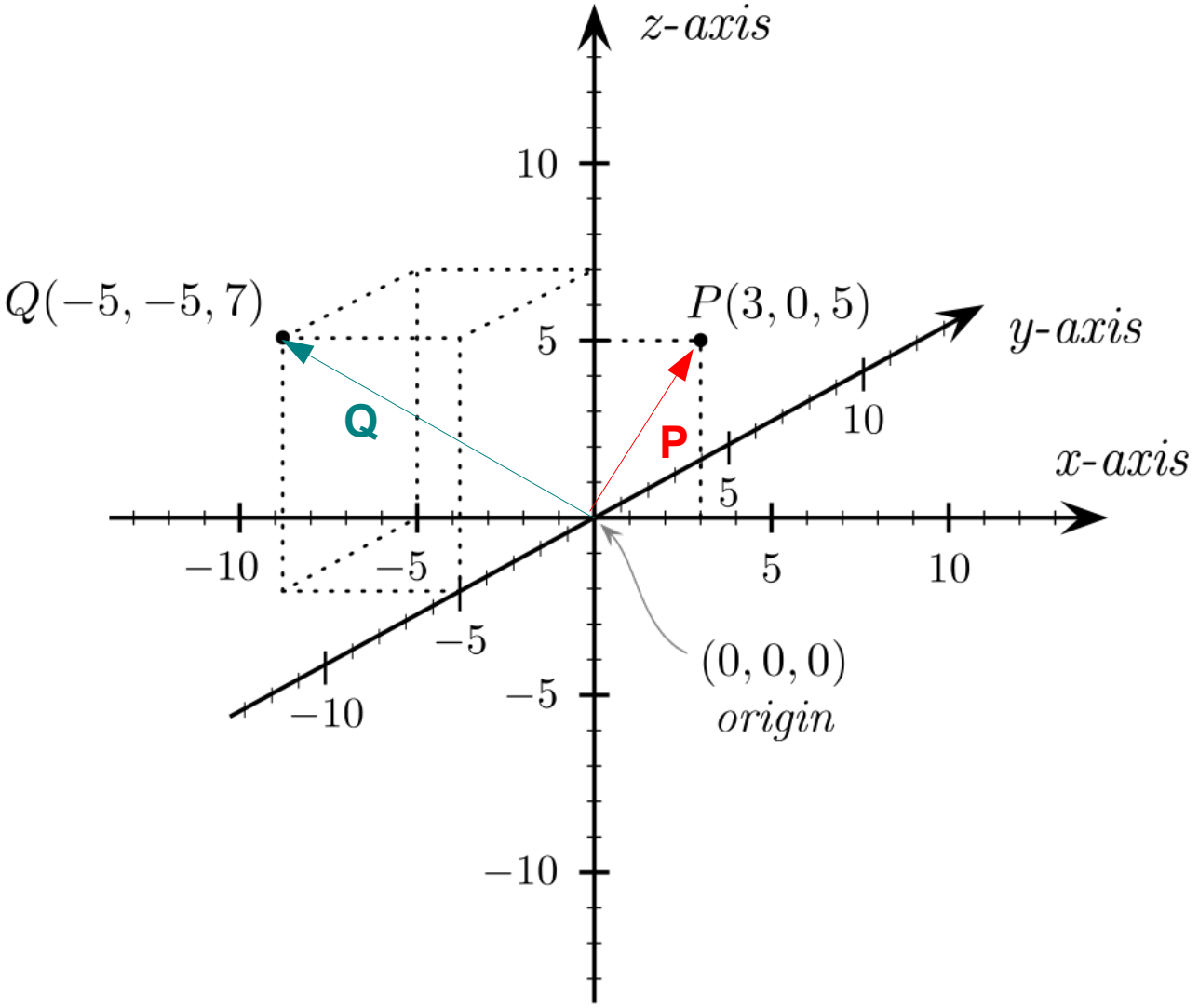


Image credit: Wikimedia Commons user 石庭豐

Points → vectors from the origin



Relative positioning of objects

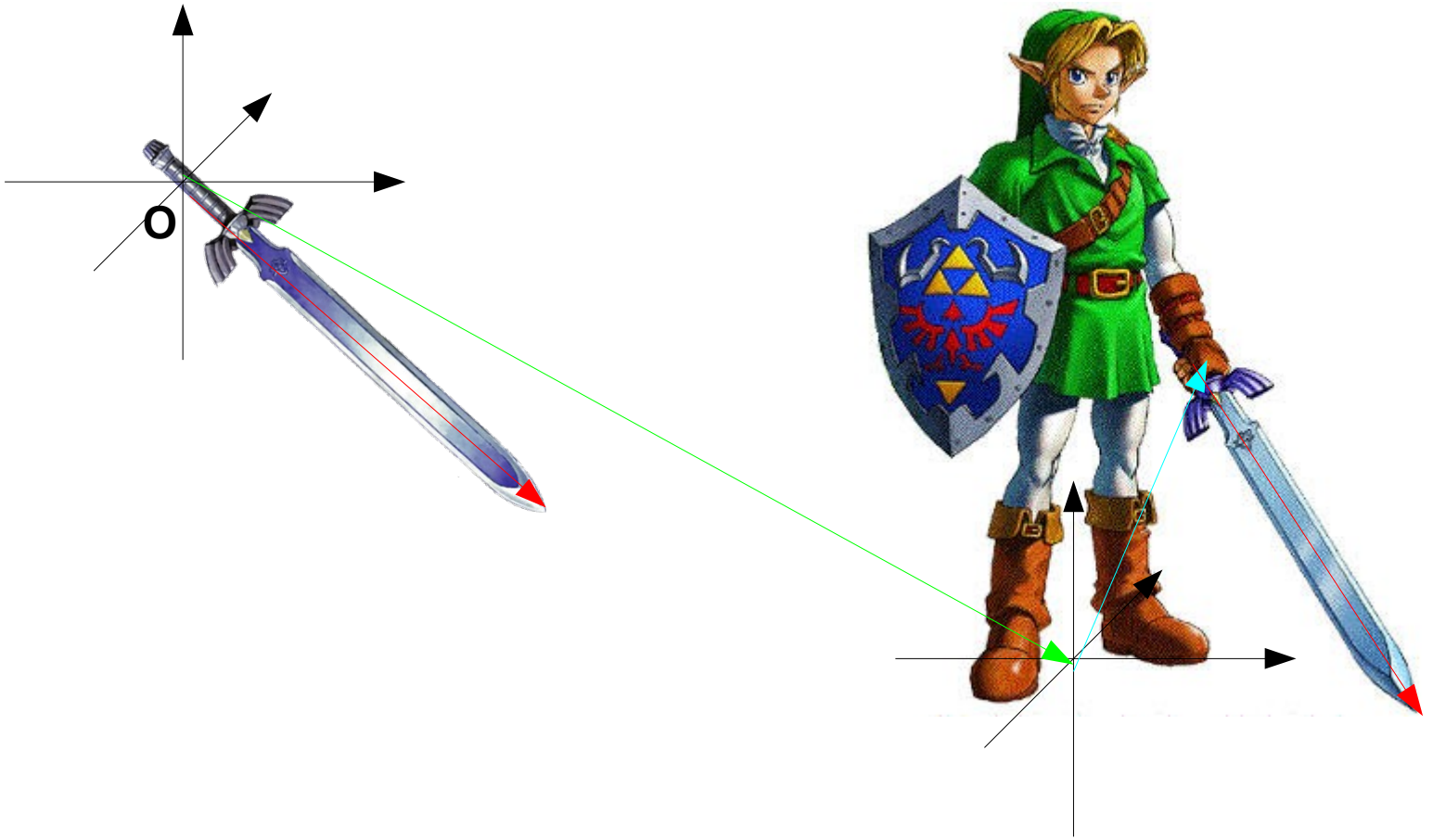


Image credits: kasuto.net, zelda.wikia.com

Vector subtraction

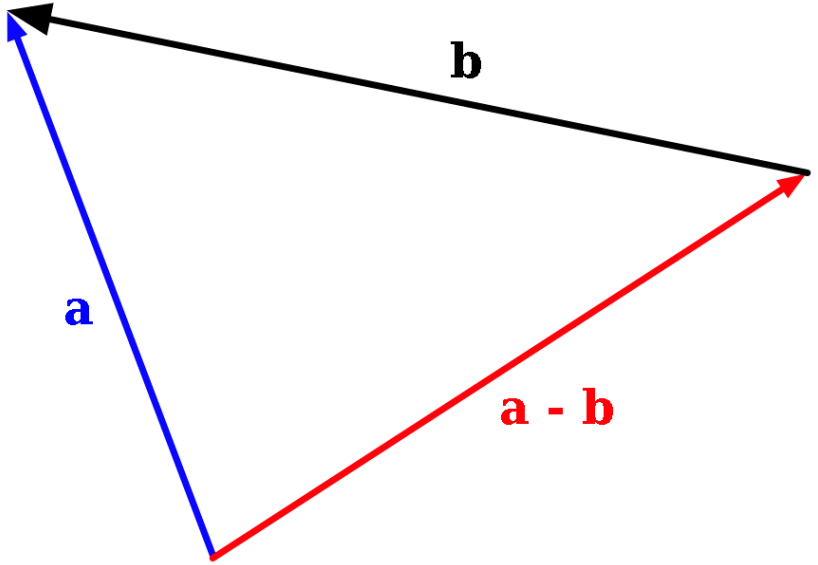
$a = (5, 6, -3)$

$b = (-1, 7, 2)$

$a - b = (5 - (-1), 6 - 7, (-3) - 2)$

$= (6, -1, -5)$

Vector subtraction



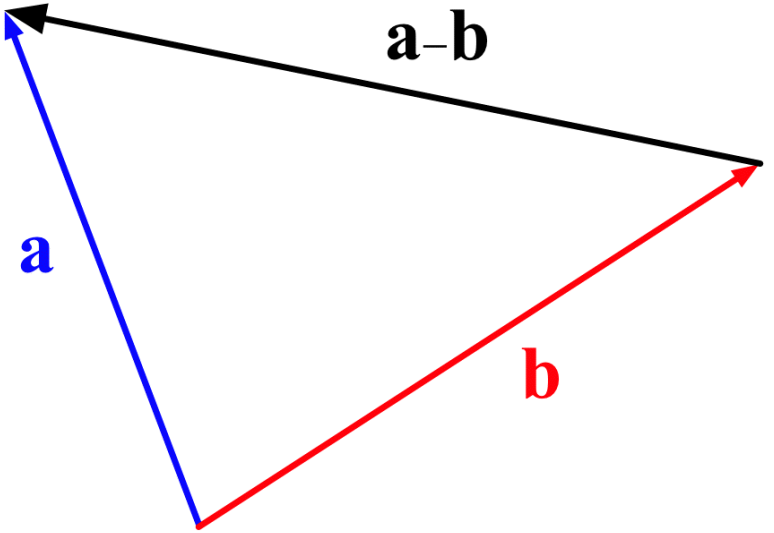


Image credit: Wikimedia Commons user bdesham

Scalar multiplication

$$a = (5, 6, -3)$$

$$5a = (5 \cdot 5, 5 \cdot 6, 5 \cdot (-3))$$
$$= (25, 30, -15)$$

Scalar multiplication

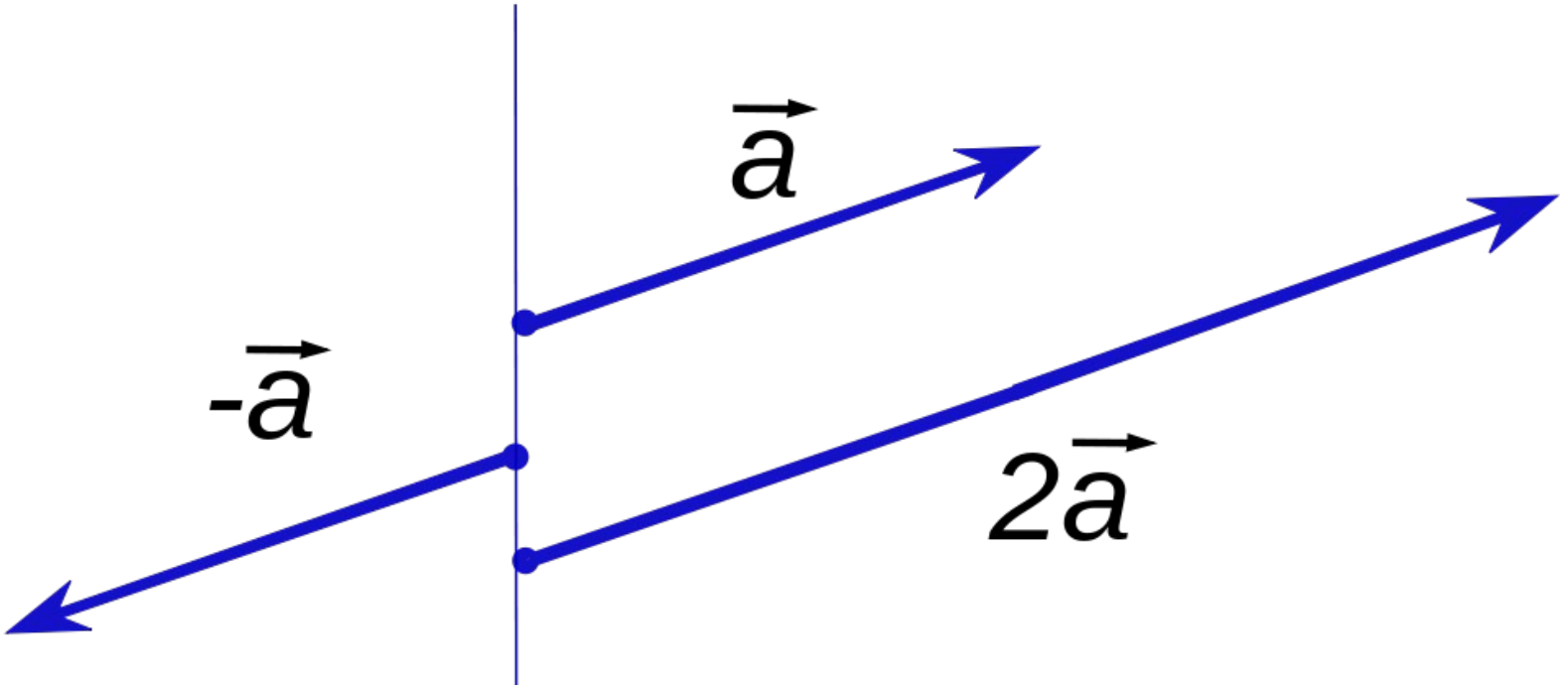


Image credit: Silly rabbit/Wikimedia Commons user Konradek

Applying force in small steps

(This is called “Euler's method for numerical integration.”
No, you don't have to remember that. But you can if you want.)

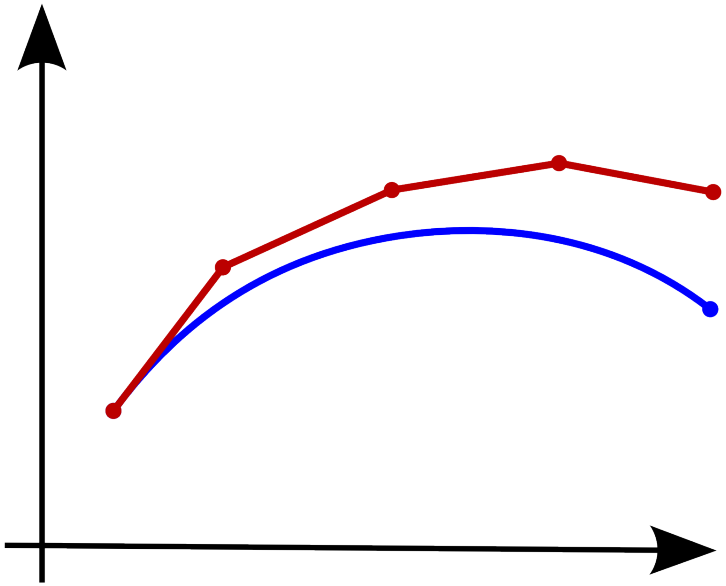


Image credit: L.P. Clerc/Wikimedia Commons user Atelierelealbe

Demo: launching snowballs

The dot product

$$a = (5, 6, -3)$$
$$b = (-1, 7, 2)$$

$$a \cdot b = 5 \cdot (-1) + 6 \cdot 7 + (-3) \cdot 2$$
$$= -5 + 42 + -6 = 31$$

An illustrative example

	Heller (R)	Reid (D)
37	Y	Y
38	Y	N
39	N	Y
40	N	N
41	N	Y
42	Y	N
43	Y	N
45	N	N
46	Y	Y
54	N	Y

Source: http://govtrack.us/

An illustrative example

	Heller (R)	Reid (D)	product
37	+1	+1	+1
38	+1	-1	-1
39	-1	+1	-1
40	-1	-1	+1
41	-1	+1	-1
42	+1	-1	-1
43	+1	-1	-1
45	-1	-1	+1
46	+1	+1	+1
54	-1	+1	-1

Total:
-2

Source: http://govtrack.us/

An illustrative example

	Boxer (D)	Feinstein (D)
37	N	N
38	N	N
39	Y	Y
40	N	N
41	Y	Y
42	N	N
43	N	N
45	N	N
46	Y	Y
54	Y	Y

Source: http://govtrack.us/

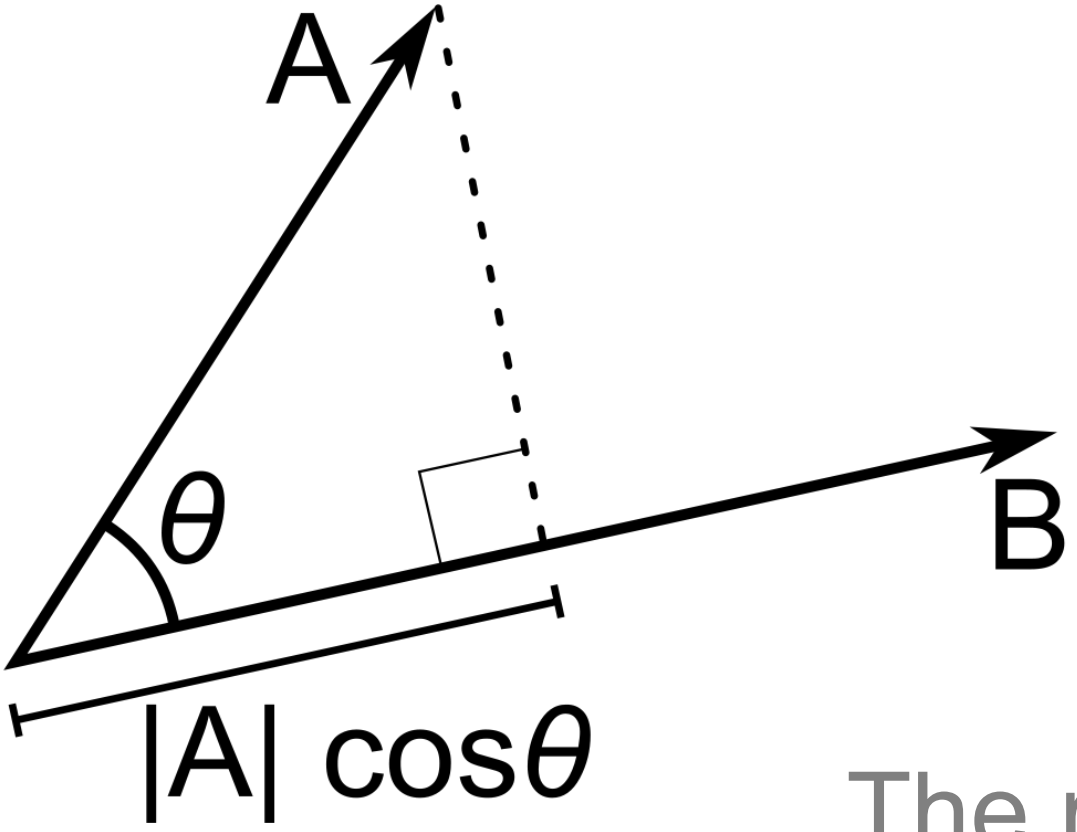
An illustrative example

	Boxer (D)	Feinstein (D)	product
37	-1	-1	+1
38	-1	-1	+1
39	+1	+1	+1
40	-1	-1	+1
41	+1	+1	+1
42	-1	-1	+1
43	-1	-1	+1
45	-1	-1	+1
46	+1	+1	+1
54	+1	+1	+1

Total:
+10 (!)

Source: http://govtrack.us/

Projecting one vector onto another



The real dot product:

$$|A|\,|\mathbf{B}|\cos\theta$$

Image credit: Wikimedia Commons user Mazin07

The cross product

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \wedge \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix}$$

Image credit: Wikimedia Commons user Ravahimanana

The cross product

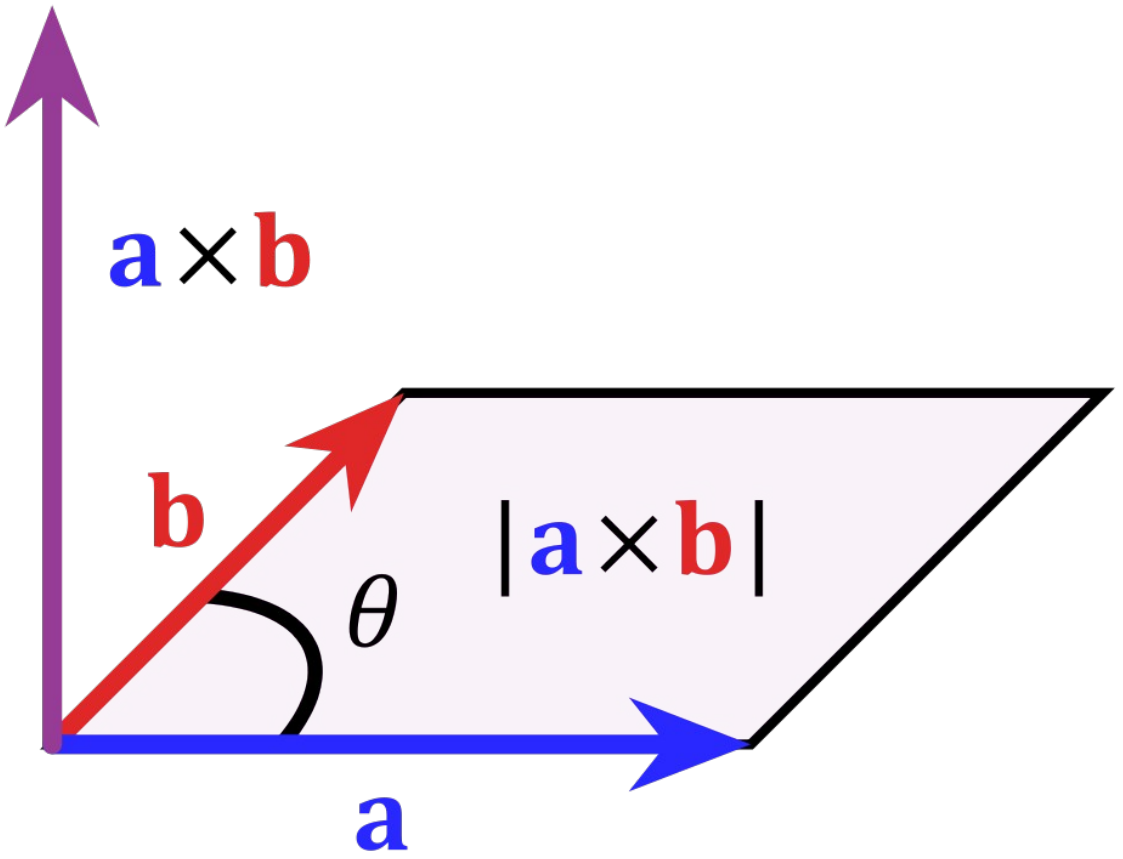


Image credit: Wikimedia Commons user Acdx

Lighting matters

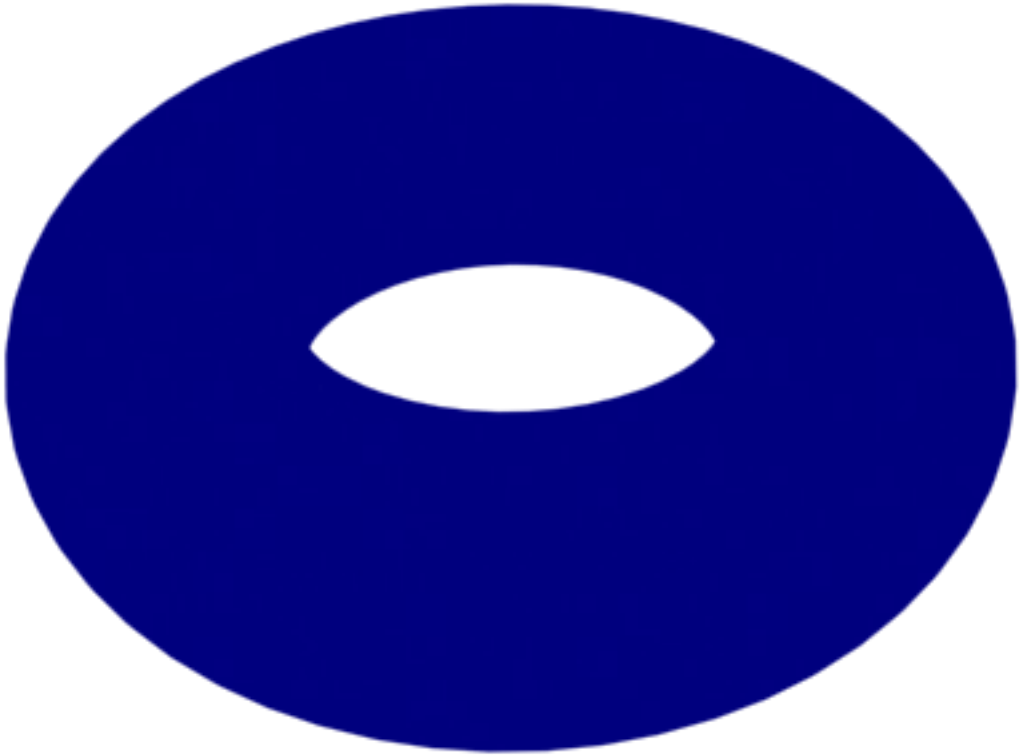


Image credit: Wikimedia Commons user Pokipsy76

Lighting matters

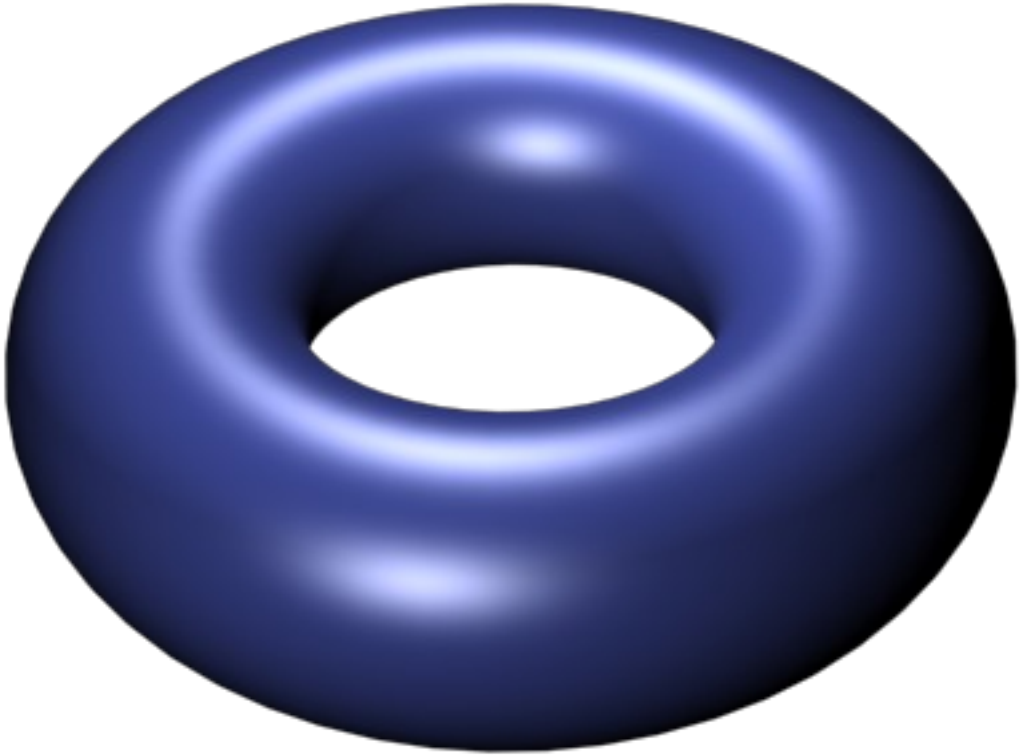
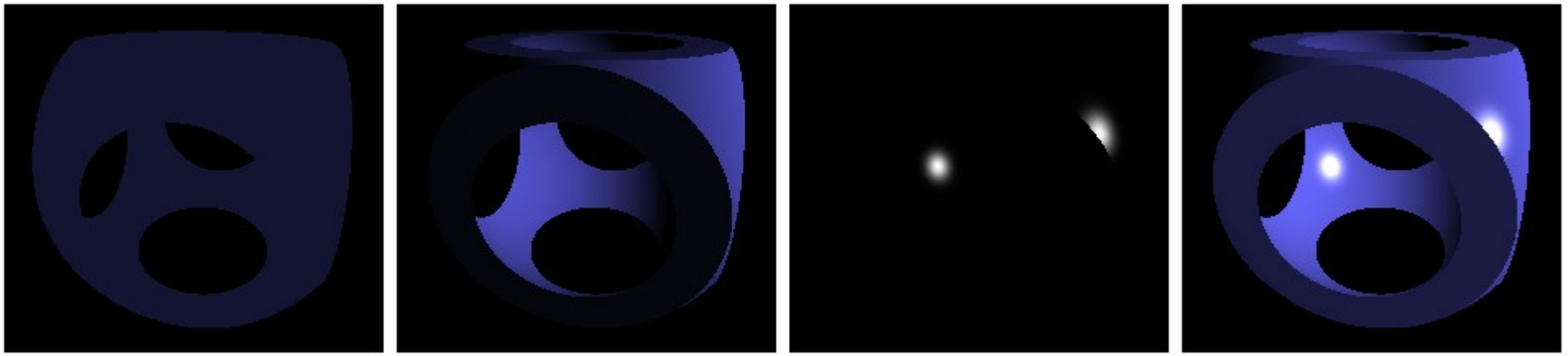


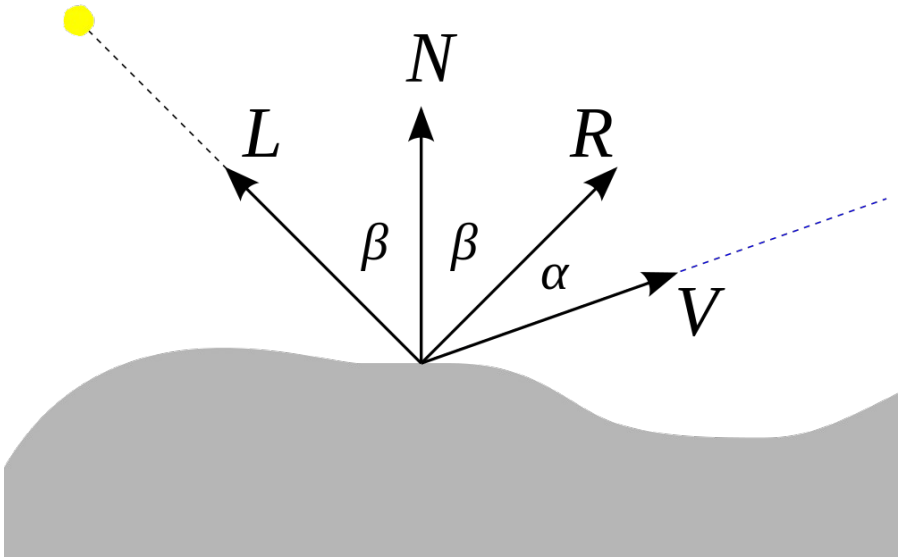
Image credit: Wikimedia Commons user Pokipsy76

Video: Phong shading

Lighting

The Phong illumination model





Ambient: constant

Diffuse: $\mathbf{L} \cdot \mathbf{N} = \cos \beta$

Specular: $(\mathbf{R} \cdot \mathbf{V})^k = (\cos \alpha)^k$

Image credit: Brad Smith/Wikimedia commons user Rainwarrior